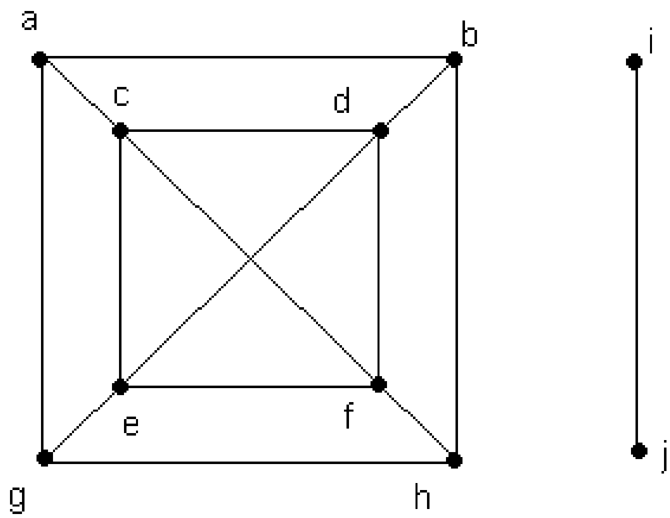
CS 70	Discrete Mathematics for CS	
Spring 2006	Vazirani	Midterm 2

PRINT your name: ______________ SID _____________ Section _____________

This is a CLOSED BOOK examination. You are allowed one single sided sheet of notes. Do all your work on the pages of this examination. Give reasons for all your answers. There are 6 problems, you have 90 minutes. Good luck!

1. (10 pts.) You are given a secret *s* which is a number modulo 11. You wish to share it among 7 people in such a way that no single person has any idea about the secret, but any two of them can reconstruct s. Suppose that the secret is $s = 5$ and you send a share $x_1 = 3$ to the first person. What shares would you send to each of the remaining 6 people? Say in a sentence how you choose the value you did. Show how person 3 to 6 would jointly recover *s*.

2. (10 pts.) What is the minimum number k of edges that must be added to this graph to make it Eulerian (i.e. so that the resulting graph has a Eulerian Tour)? Prove that k - 1 edges are not sufficient.



3. (10 pts.) A balanced bridge hand has four cards from some suit and three from each of the rest (a bridge hand consists of 13 cards). How many balanced bridge hands are there? Remember to explain how you derived your answer.

4. (20 pts.) A polynomial Q of degree at most 9 is picked uniformly at random among all polynomials of degree (at most) 9 modulo 13.

(a) What is the sample space, and what is the probability of each sample point.

(b) Let A be the event that $Q(1) = 5$ and $Q(2) = 7$. What is the probability of A. Your answer should be a rational number.

(c) Let B be the event that $Q(3) = 5$ and $Q(4) = 7$. What is $P[B|A]$, the conditional probability of B given A?

(d) Are A and B independent events? Remember to justify your answer.

5. (15 pts.) Conditional Probability

The Cal Bears are playing Stanford in a 2-out-of-3 series, i.e. they play games until one team wins a total of two games. The probability that the Bears win the first game is ½. For subsequent games, the probability of winning depends on the outcome of the preceding game; the team is energized by victory and demoralized by defeat. If the Bears win a game, then they have a 2/3 chance of winning the next game. On the other hand, if the Bears lose, they have only a 1/3 chance of winning the next game.

(a) What is the probability that the Bears win the 2-out-of-3 series given that they win the first game?

(b) What is the probability that the Bears won the first game, given that they won the series?

6. (10 pts.) We choose k elements out of {1, 2, ..., n} without replacement. What is the probability that the sequence of elements chosen is strictly increasing. For example, the sequence 1,5,20,37 is an increasing sequence, whereas the sequence 1,5,20,15,37 is not.