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CS 70

Discrete Mathematics for CS

Spring 2006

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Midterm I

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PRINT your name: \_\_\_\_\_ SID \_\_\_\_\_ SECTION \_\_\_\_\_

This is a CLOSED BOOK examination. You are allowed one single sided sheet of notes. The use of calculators is not permitted. Do all your work on the pages of this examination. Give reasons for all your answers. Good Luck!

**1. (25 pts.) Quantifiers and Proofs**

For each of the following, indicate whether the two statements are equivalent.

(a)  $\forall x (P(x) \vee Q(x))$  and  $(\forall x P(x)) \vee (\forall x Q(x))$ .

(b)  $\forall x (P(x) \wedge Q(x))$  and  $(\forall x P(x)) \wedge (\forall x Q(x))$ .

(c)  $\forall x \neg P(x)$  and  $\neg \forall x P(x)$ .

The famous mathematician Fermat believed (falsely) that he had proved the following theorem:  
 $\forall n \in \mathbf{N}$  if  $n = 2^{2^k} + 1$ ,  $k \in \mathbf{N}$  then  $n$  is prime.

(d) You have decided to prove this statement by contraposition. What statement would you be trying to prove.

(e) You get confused and try to prove the converse instead. What statement would you prove in this case.

**2. (15 pts.) Induction**

Suppose you know that  $P(1)$  is true, and that  $\forall k \geq 1, P(k) \Rightarrow P(2k)$ . Use induction to show that  $P(n)$  is true whenever  $n$  is a power of 2.

**3. (10 pts.)**

Prove that  $\forall n \in \mathbf{N} \ 14^n - 1$  is divisible by 13.

**4. (15 pts.) Stable Marriage**

In a particular instance of the stable marriage problem with  $n$  men and  $n$  women, it turns out that there are exactly three distinct stable matchings,  $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$ . Also, each woman  $W$  has a different partner in the three matchings. Therefore each woman has a clear preference ordering of the three matchings (according to the ranking of their partners in their preference list). Now, suppose for woman  $W_1$ , this order is  $\mathcal{M}_1 > \mathcal{M}_2 > \mathcal{M}_3$ . True or false: every woman has the same preference ordering  $\mathcal{M}_1 > \mathcal{M}_2 > \mathcal{M}_3$ . Justify your answer.

**5. (10 pts.) Modular Arithmetic**

Show that if  $a \equiv b \pmod{m}$ , then for  $c \in \mathbf{N}$ ,  $ac \equiv bc \pmod{mc}$ .

**6. (10 pts.) Modular Division**

Evaluate  $(6/5)^{25} \pmod{11}$ .

**7. (15 pts.) Chinese Remainder Theorem**

Calculate  $31^{4,801} \pmod{35}$ , using the Chinese remainder theorem. Show your work.

Hint:  $3^6 \equiv 1 \pmod{7}$ .