Student Name:

## Class Account Username:

## Instructions: Read them carefully!

This exam begins at 7:10 pm and ends at 9:00 pm. You must turn your exam in when time is announced or risk not having it accepted.

Make sure you fill in your name and the above information, and that you sign below. Anonymous tests will not be graded.

Write legibly. If the person grading the test cannot read something, he/she will simply assume that you meant the illegible portion as a note to yourself and they will ignore it. If you lose points because part of your answer could not be read, you will not be given the opportunity to explain what it says.

Be clear and concise. The answers to most questions should be short. If you find yourself writing an excessively long response, you may want to think more carefully about the question. Long rambling answers generally get fewer points that short ones do because there are more opportunities to mark something wrong.

You may use two pages of notes while taking the exam. You may not ask questions of other students, look at another student's exam, use a textbook, use a phone or calculator, or seek any other form of assistance. In summary: do not cheat. Persons caught cheating will be subject to disciplinary action.

Do not ask questions during the exam. Most questions are unnecessary and they disturb other students. Figuring out what the exam question is asking is part of the test. If you think you have to make some unusual assumption to answer a problem, note what that assumption is on the test.

## I have read these instructions, I understand them, and I will follow them.

## Your Signature:

## Date:

## Student ID:

1. Please fill in each of the blanks with an appropriate answer.

The Euler integration scheme tends to cause simulations to "blow up." The implicit version of this scheme, known as $\qquad$ , is much more stable but has a tendency to damp motions artificially.

The inverse kinematics algorithm we discussed in class involve numerical root finding for a set of nonlinear equations and can be solved using a variation of $\qquad$ method.
$\qquad$ motion capture systems record the position of reflective markers on the subject.

The cross-product of the tangent vectors of an parametric surface generally can be used to compute the surface $\qquad$ _.

When representing $\qquad$ in 3D using homogenized coordinates, the fourth coordinate (i.e. "w") will be zero.

Temporal anti-aliasing is often called $\qquad$ .

The radiosity method builds on the assumption that all surfaces in a scene are
$\qquad$ reflectors.

The final gather often is the most time consuming step of the rendering method known as
$\qquad$ .

Computing the form factors between surface patches often is the most time consuming step of the rendering method.
$\qquad$
$\qquad$ approximates global illumination by making diffuse shading proportional to the un-occluded area over a surface.

Catmull-Clack subdivision surfaces are a generalization of uniform, cubic, tensor-product
$\qquad$ to irregular meshes.

NURBS surfaces are a generalization of uniform, polynomial, tensor-product to be non-uniform and rational-polynomial.

In Catmull-Clark subdivision, the number of extraordinary points $\qquad$ during the third round of subdivision.

Under linear perspective projection, straight lines always appear as $\qquad$ in the resulting images.

Violet spectral colors appear at the $\qquad$ end of the visible spectrum.

The $\qquad$ of an orthonormal matrix is equal to its transpose.

The $\qquad$ parameterization of 3D rotations uses a hypersphere in 4-dimensional space.

The " $P$ " in BSP-Tree stands for $\qquad$ .
is a special case of a simple point light source where the viewer is infinitely far away.
is used to change the surface normal vectors when shading an object so that it appears to have surface bumps and other geometric details.

When two curve segments join at a point and both curves approach that point with tangent vectors of equal direction and magnitude, the joining is said to be $\qquad$ continuous.

When two curve segments join at a point and both curves approach that point with non-zero parallel tangent vectors from opposite directions, the joining is said to be $\qquad$ continuous.

Steradians are the dimensionless units used to measure $\qquad$ .

When the viewpoint used to generate a ray-traced image changes, updating the solution for the new viewpoint requires recomputing $\qquad$ .
2. Answer the following questions with True (T) or False (F)
$\qquad$ Cloth can be modeled in a perfectly realistic way using a collection of particles attached by springs.
$\qquad$ The pseudo inverse of a matrix can be computed using the Degenerate Value Decomposition (DVD) algorithm.
$\qquad$ The Jacobian of a valid kinematic system will never be invertible.
$\qquad$ Radiance grows with the square of distance along a straight line in free space.
$\qquad$ Both the cones and rods in the human eye have constant spectral response functions.
$\qquad$ Under perspective projection, all sets of parallel lines will remain parallel in the image.

Any set of non-intersecting polygons can be sorted in front-to-back order from any perspective.

Shining an ultraviolet light on scorpions can create a situation where ultraviolet photons are adsorbed and lower energy photons are then emitted in the visible part of the spectrum.

The force exerted by a spring with non-zero rest length is given by a function that is linear in terms of the endpoint locations.

A light field records the light passing through a single point in space in all directions.

In a kinematic skeleton, every body must have exactly one outboard joint.

Apple retina displays have a dynamic range exactly equal to that of the human eye.

A rotation matrix always has determinant of -1 .

Motion capture systems typically run at rates between 20 and 30 samples per second.

The sky is blue because photons in the red-orange parts of the spectrum are absorbed by nitrogen atoms.
3. The diagram below shows control points for a curve made by joining two cubic Bezier segments. However control point \#5 has been removed. Indicate location(s) where \#5 may be placed to achieve C1 continuity and where it may be placed to achieve G1 continuity. Clearly label your diagram.

6 points

## (4)

(3)


4. Here is a piece of mesh. Draw the result of applying one iteration of Catmull-Clark subdivision. Then circle all vertices (both original and the new ones you added) that are extraordinary. Note: I am only interested in the topology of your answer. The square boundary lines are part of the mesh.

5. Below are three $4 \times 4$ homogenized transformation matrices. Describe what they will do. 6 points

$$
\left[\begin{array}{llll}
3 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & 6
\end{array}\right] \quad\left[\begin{array}{llll}
6 & 0 & 0 & 0 \\
0 & 6 & 0 & 0 \\
0 & 0 & 6 & 0 \\
0 & 0 & 0 & 2
\end{array}\right]\left[\begin{array}{cccc}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The first one will: $\qquad$

The second one will: $\qquad$

The third one will: $\qquad$
6. Draw the convex hulls of the each of the shapes shown below.

6 points

7. The diagram below is the control polygon for a Bezier curve segment. Draw the curve and show how de Casteljau's algorithm can be used to subdivide the curve into two halves. Make sure your drawing is geometrically reasonable and shows correct curve tangents for the the beginning, middle, and end of each segment.

6 points

8. Given a rotation matrix, how would you determine the axis that it rotates around?
9. In the diagram below of a light source, a clear glass ball, and a diffuse surface, draw lines illustrating the path traveled by light to form a refraction caustic on the surface.

3 points

10. Write out an explicit equation for a sphere. Be sure to specify the range of your parametric variables to exactly cover the sphere once.

4 points
11. Given three points, $A, B$, and $C$, write out an implicit equation for the plane in 3D that contains all three points. You may assume the points are distinct and not co-linear.
12. Consider the following equation and diagram:
$L_{S}\left(\mathbf{x}, \mathbf{x}^{\prime}\right)=\delta\left(\mathbf{x}, \mathbf{x}^{\prime}\right)\left[E\left(\mathbf{x}, \mathbf{x}^{\prime}\right)+\int_{S} \rho_{x^{\prime}}\left(\mathbf{x}, \mathbf{x}^{\prime \prime}\right) L_{S}\left(\mathbf{x}^{\prime}, \mathbf{x}^{\prime \prime}\right) \frac{\cos \left(\theta^{\prime}\right) \cos \left(\theta^{\prime \prime}\right)}{\left\|\mathbf{x}^{\prime}-\mathbf{x}^{\prime \prime}\right\|^{2}} \mathrm{~d} \mathbf{x}^{\prime \prime}\right]$


Explain what effects each of the following is responsible for.
10 points
$\delta\left(\mathbf{x}, \mathbf{x}^{\prime}\right)$
$E\left(\mathbf{x}, \mathbf{x}^{\prime}\right)$
$\left\|\mathbf{x}^{\prime}-\mathbf{x}^{\prime \prime}\right\|^{2}$
$\cos \left(\theta^{\prime}\right)$
$\rho_{x^{\prime}}\left(\mathbf{x}, \mathbf{x}^{\prime \prime}\right)$
13. If a surface is defined explicitly by the function $\mathbf{x}=f(u, v)$, write out the equation you would use to compute the surface's normal at some point. 4 points
14. Consider this diagram showing a four-joint arm in 2D where each joint is a simple pin joint and the base is fixed in space.


If we are solving an IK problem to place the tip of the arm (the black dot) at a particular location, what is the size of the Jacobian matrix we will be working with?

3 points

Draw any one configuration of the arm where two columns of the Jacobian will be parallel vectors.

5 points
15. Name two types of useful image effects that would require shooting more than one ray per pixel in a ray-tracer.

4 points
16. The following are the response curves for the cones in the human eye. Which type of cone is most sensitive to red light?

2 point

17. Consider the diagram below. A location has been marked on the surface. Indicate a viewer position such that a viewer looking at the surface from that position would see a specular highlight on the surface at the location marked with a small square/diamond. 3 points

18. On the diagram below, draw the springs that should be added to provide some resistance to shearing.

Extra Credit +5 points

Given a plane and a sphere:
Plane:
$\hat{\mathbf{n}} \cdot \mathbf{x}+f=0$
Sphere:

$$
\|\mathbf{c}-\mathbf{x}\|-r=0
$$

Write out a simple mathematical expression that will give the point on the sphere which is furthest away from the plane. (You may not include conditional statements.)

When is this point not unique?

When the point is not unique, what will your expression produce?

Your answers must be neat and clear. No points will be awarded for imprecise answers. Your answers should be in the form a simple mathematical expressions that you have drawn a box around. Do not attempt this question until you have completed the rest of the exam! There will be no partial credit for this question. Use the below space and back of the page to work out your answers, and do not clutter up the above space with anything other than your final answers.

