Instructions: Read them carefully!

The exam begins at 1:10pm and ends at 2:30pm. You must turn your exam in when time is announced or risk not having it accepted.

Make sure you fill in your name and the above information, and that you sign below. Anonymous tests will not be graded.

Write legibly. If the person grading the test cannot read something, s/he will simply assume that you meant the illegible portion as a note to yourself and they will ignore it. If you lose points because part of your answer could not be read, you will not be given the opportunity to explain what it says.

Be clear and concise. The answers to most questions should be short. If you find yourself writing an excessively long response, you may want to think more carefully about the question. Long rambling answers generally get fewer points that short ones do because there are more opportunities to mark something wrong.

You may use one page of notes while taking the exam. You may not ask questions of other students, look at another student’s exam, use a textbook, use a phone or calculator, or seek any other form of assistance. In summary: do not cheat. Persons caught cheating will be subject to disciplinary action.

Do not ask questions during the exam. Most questions are unnecessary and they disturb other students. Figuring out what the exam question is asking is part of the test. If you think you have to make some unusual assumption to answer a problem, note what that assumption is on the test.

I have read these instructions, I understand them, and I will follow them.

Your Signature: ____________________________

Date: ______________________________________

Student ID: ________________________________

Total Points: 227 + 8 You Scored: _____________ + _______________
1. Please fill in each of the blanks with an appropriate answer. 2 points each blank, 88 Total

True or False: The dynamic range of intensities that can be displayed on a smartphone screen is larger than that of the human eye. 

True or False: Humans are good at judging the relative brightness of two squares on a screen when there is a bright red band of color separating them.

Visible light falls roughly in the range of nanometers at the BLUEnanometers at the RED end of the spectrum to nanometers at the RED end.

Fill in the two missing colors in order: Red, Orange, , Green, , Indigo, Violet.

 colors consist of a single wavelength (or very narrow band of wavelengths) of light.

The sensitivity curves for the and cones have a substantial amount of overlap.

The cones are concentrated in the part of the human eye.

The part of the human eye mainly contains rods.

In Tobi Pfaff’s lecture last week on clipping and hidden-surface removal, he included a video or what animal?

True or False: A Z-buffer stores the scene depth at each pixel as an 8-bit integer.

True or False: Any color humans can see can be reproduced using any two distinctly colored light sources.

 are distinct materials that appear to be the same color under some specific lighting.
_________________ is the main phenomenon that makes milk appear white.

True of False: The BRDF describes how much light coming in from one incoming direction goes out in another outgoing direction. ________________

True of False: Smeagol’s law describes how refractive materials behave. ________________

True of False: The exponent in the Phong shading model controls how bright a material appears. ________________

A _________________ can be thought of as a point light source located “at infinity”.

True or False: Mach banding tends to underemphasize edges so that shading may appear excessively smooth. ________________

True or False: Flat shading is named after the French computer graphics researcher Jon-Paul Flat. ________________

True or False: Rotation is a linear transformation. ________________

The rows and columns of an arbitrary rotation matrix are always ________________.

True of False: Matrix multiplication is associative. ________________

True of False: Homogenized coordinates are needed to allow rotation to be expressed as matrix multiplication. ________________

Of the various methods discussed in class for representing rotations, the method of ________________ is least appropriate for interpolation due to singularities which include gimbal lock.

Rotation matrices in 3D generally have ________________ (number) of real eigenvalues.
In ray tracing, sending multiple rays through a given pixel and averaging the results is called __________________, and it is used most often for __________________.

The implicit formula for a sphere is ________________________________.

True or False: BSP Trees can be used to accelerate ray intersection tests for complex scenes. ______________

Under *orthographic* projection straight lines will always appear as ________________.

Under *orthographic* projection spheres will generally appear as ________________.

Which points in a linear perspective image are vanishing points for some set of parallel lines? ________________

A key feature of Bresenham’s line drawing algorithm is that it uses only __________________ arithmetic.

A __________________ stores depth/distance values and can be used for shadow computation when rendering a scene.

The __________________ breaks a matrix A into A = U S Transpose(V) where U and V are orthonormal and S is diagonal.

True or False: A BSP tree can be used to quickly sort polygons in front-to-back order. ______________

True or False: The color of some objects results from wave interference. ________________

True or False: Rotation about an arbitrary axis in 3D requires three separate transformation matrices. ________________

True or False: Systems like OpenGL typically convert everything to simple quadrilaterals before rendering. ________________
2. If you have two unit vectors, A and B, then we can write the dot and cross products as \( A \cdot B \) and \( A \times B \), respectively. Simplify the following expressions:  

\[
(B \times A) \times (A \times B) = \\
A \times (B \times (A \times B)) = \\
B \times (A \times (B \times (A \times B))) =
\]

3. You have a sphere centered at \([3,3,3]\) with radius 4, and a ray from \([10,10,10]\) in the direction \([-1,-1,-1]\). Write the implicit equation for the sphere, the parametric equation for the ray, and compute the coordinates of the intersection points. \textit{Be neat and clear!}

\textit{Sphere equation:}

\textit{Ray equation:}

\textit{Intersections at =}

4. Write out a 3x3 transformation matrix that will rotate the vectors \([\sqrt{1/2}, \sqrt{1/2},0]\), \([0,0,1]\), and \([\sqrt{1/2},-\sqrt{1/2},0]\) to align with the X, Y, Z axes respectively.

5. Circle the types of transformations that to be expressed in matrix form do NOT require homogenized coordinates.

\begin{tabular}{l}
Translation \\
Rotation \\
Shear \\
Scale \\
Perspective \\
\end{tabular}
6. Draw the single **convex hull** that encloses all four shapes shown:  

7. Draw the single **axis-aligned bounding box** that encloses all four shapes shown:
8. One of the diagrams below shows a cube under orthographic projection, the other under perspective projection. Label which is which.  

2 points

9. Given a rotation encoded as an exponential map with the vector shown, write out a vector that express the inverse rotation. (units are radians)  

3 points

\[ [1, 0, 0] \]

10. Given a rotation encoded as a quaternion, in general how is the rotation changed when the only the imaginary part is negated?  

3 points

11. Write down plausible RGB values for the following materials:  

6 points

Glossy Metallic Magenta
\[ Kd = \]
\[ Ks = \]

Glossy Plastic Yellow
\[ Kd = \]
\[ Ks = \]

Flat Cyan
\[ Kd = \]
\[ Ks = \]
12. If shading a point at the origin with normal [0,1,0] and Kd=[0.2 ,0.4 ,0.6], where the light is located at [0,4,3] with intensity 20 (white), and the eye located at [27,91,17], compute the RGB value of the **diffuse** lighting term.  

10 points

13. Circle the 3D homogenized matrix that would scale objects by 2x.  

4 points

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 2 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1/2 \\
\end{bmatrix}
\]

14. Let f(x,y) be a scalar function on the plane. Write out the expression for the **downward** pointing gradient.  

3 points

15. Draw an example of 3 polygons that do not intersect, but that cannot be sorted in front-to-back order from the viewer’s perspective.  

1 point
16. The following line segments will be inserted into a BSP Tree in the order indicated. As discussed in class, the lines themselves will be used to define the split planes. The numbers are on the positive side of each line. The negative-side children should be on the left of your tree and the positive-side on the right.

Diagram the resulting tree below. If needed, show where line segments need to be split by marking on the above figure. Also, indicate the names of the split parts by writing labels on the figure above. (For example, if there were a segment 11 and it was to be split, you would draw a mark showing where it would be split and label the resulting pieces 11a and 11b.)

List the back-to-front traversal order that would result for the location indicated by the viewer icon (the star).
17. Write out the 3x3 matrix for a rotation about the Z axis. Now write out a 3x3 matrix for a rotation about the Z axis that would result if we did rotations clockwise instead of counterclockwise.  

6 points

18. Imagine that you have a RGB monitor where the wires have been swapped so that the red, green, and blue outputs from the computer have been respectively attached to the red, blue, and green inputs on the monitor. When one attempts to display the following colors, what colors will actually appear on the screen?  

8 points

Cyan

Magenta

Yellow

Red

Green

Blue

Black

White
19. On the figure below write the appropriate letter in each of the blanks to label the diagram properly. Some of the letters are just there to confuse you.

- A: Gimbal lock
- B: Top clipping plane distance
- C: End of all the things
- D: Center of projection
- E: Origin
- F: Look-at vector
- G: Near clipping plane distance
- H: View horizon
- I: Bottom clipping plane distance
- J: Distance to image plane
- K: Left clipping plane distance
- L: View plane tangent
- M: View plane normal
- N: View down vector
- O: Far clipping plane distance
- P: View up vector
20. Write out the transformation steps discussed in class for a perspective camera. It may help to refer to the previous question. 12 points
Given:

A sphere defined by 

$$\| \mathbf{x} - \mathbf{c} \|^2 - r^2 = 0$$

and a normal direction

$$\hat{n}$$

Write out an implicit equation for the largest circle on the sphere that lives in a plane normal to the specified normal direction.

When will the circle be undefined?

Your answer must be neat and clear, written out in the boxes. No points will be awarded for imprecise answers. You must get all parts right to earn any credit. (i.e. all or nothing) Do not attempt this question until you have completed the rest of the exam!