Instructions: Read them carefully!

The exam begins at 3:10pm and ends at 6:00pm. You must turn your exam in when time is announced or risk not having it accepted.

Make sure you fill in your name and the above information, and that you sign below. Anonymous tests will not be graded.

Write legibly. If the person grading the test cannot read something, he/she will simply assume that you meant the illegible portion as a note to yourself and they will ignore it. If you lose points because part of your answer could not be read, you will not be given the opportunity to explain what it says.

You may use two pages of notes while taking the exam. You may not ask questions of other students, look at another student’s exam, use a textbook, use a phone or calculator, or seek any other form of assistance. In summary: do not cheat. Persons caught cheating will be subject to disciplinary action.

Do not ask questions during the exam. Most questions are unnecessary and they disturb other students. Figuring out what the exam question is asking is part of the test. If you think you have to make some unusual assumption to answer a problem, note what that assumption is on the test.

The answers to most questions should be short. If you find yourself writing an excessively long response, you may want to think more carefully about the question.

I have read these instructions, I understand them, and I will follow them.

Your Signature: __________________________________________

Date: __________________________________________
1. Answer the following questions with True (T) or False (F)  

   _____ Motion blur is essentially temporal anti-aliasing.

   _____ One of the inverse kinematics algorithms discussed in class involves numerical root finding on a set of nonlinear equations and can be solved using a variation of Newton’s method.

   _____ Optical motion capture systems record the position of reflective markers on the subject using ultraviolet cameras.

   _____ Catmull-Clack subdivision surfaces are a generalization of uniform, cubic, tensor-product b-splines to irregular meshes.

   _____ Spring-and-mass systems can be used to animate a wide range of objects including hair and cloth.

   _____ Although they are interesting, particles systems have few practical uses.

   _____ Euler integration is only stable for time steps of 0.001 seconds or less.

   _____ It is not possible to reuse or combine recorded motion capture data.

   _____ Feet sliding on the ground is a common problem for motion capture data, but it is very hard to notice so in practice people ignore the problem.

   _____ In a kinematic skeleton, every body must have exactly one inboard joint, but may have many outboard joints.

   _____ Given a proper kinematic skeleton, there will always be some finite number of solutions for a given IK problem.

   _____ NURBS surfaces are a generalization of uniform, polynomial, tensor-product b-splines to be non-uniform and rational-polynomial.

   _____ Using implicit functions for “morphing” shapes is convenient because it becomes easy to handle topological changes.

   _____ The idea of splines as a modeling tool is based on thin strips of wood, metal, or plastic that were once commonly used by draftsmen.
If a curve is not $G^i$ continuous then it's parameterization cannot be $C^i$.

Modern LCD displays have a dynamic range about 10x that of the human eye.

All linear transformations can be decomposed into a series of “primitive” transformations.

The axis-angle representation of rotations, also called existential maps, encode rotations as points inside a ball of $9\pi$ radius.

A rotation matrix never has a negative determinant.

The near and far clipping planes serve no use other than a slight speed up from not drawing some objects.

The sky is blue because the water vapor in the atmosphere absorbs the ultraviolet (a.k.a. black) light from space and reemits it as visible blue light.

Any set of non-intersecting polygons can be sorted in front-to-back order using a BSP tree without having to split any of the polygons.

The cones in the human eye are each sensitive to a part of the visual spectrum that does not overlap with the region that the rods are sensitive too.

The Jacobian of a valid kinematic system will always be invertible.

Mach-banding effects may often occur when rendering adjacent regions of constant color that are slightly different.

An image of a daytime scene can be made to look as if it were a nighttime scene by multiplying the blue channel values by 0.525.

Under perspective projection, all sets of parallel lines will remain parallel in the image.

The number of rays used to ray trace a scene will typically be logarithmic in the bounce depth.

Computing motion blurred images of fast-moving objects is typically quite expensive compared to non-motion blurred images of the same scene.
To avoid artifacts, motion capture systems record data at exactly the same frame rate as the final output animation.

Overly large time-steps can cause a spring and mass simulation to go unstable.

Inter-particle forces, like gravitational attraction or collisions, require $n^2$ computation and there is no known way to reduce this requirement.

Cubic Bezier curves are specified using the positions and first derivatives at the beginning and end of each segment.

When representing points in 3D space using homogenized coordinates, the fourth coordinate (i.e. “w”) will be zero.

Motion graphs are directed acyclic graphs.

A kinematic structure with no loop joints is a directed acyclic graph.

All significant data structures are really just directed acyclic graphs.

When two curve segments join at a point and both curves approach that point with non-zero parallel tangent vectors from opposite directions, the joining is said to be discontinuous.

Under perspective projection, each set of parallel lines will appear to converge to a point. Except for lines parallel to the viewing plane, which remain parallel in the image.

Particle systems were invented in the late 70’s and although they were useful in early computer graphics, they are no longer a commonly used tool.

The development of polynomial b-spline curves was motivated by the need to represent circles exactly.

Large crowds of animated characters can be animated using a motion graphs that are driven by rules that model the characters’ behavior.
2. Here is a piece of mesh. Draw the result of applying one iteration of Catmull-Clark subdivision. Then circle all vertices (both original and the new ones you added) that are extraordinary. *Note that only the topology of your answer is relevant.*  

16 points
3. Below are three 4x4 homogenized transformation matrices. When applied to an object, what would each of them do?  

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 2 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

The first one will: ____________________________________________________________

The second one will: _________________________________________________________

The third one will: ___________________________________________________________

4. You have a parametric surface defined by some function, \( f(x) \). Write out an expression that you would use to compute the surface normal at some point.  

6 points

5. The two images below show two 12 point versions of the letter “A” that have been significantly enlarged. Concisely explain the most likely difference between the two fonts used in the images.  

4 points
6. The diagram below is the control polygon for a Bezier curve segment. Draw the control polygon for a second segment that connects to the first segment at vertex #4 with $C^1$ continuity. Draw both segments of the resulting curve. Make sure your drawing is geometrically reasonable and shows correct curve tangents for the beginning, middle, and end of each segment.

![Bezier Curve Diagram]

7. Given a 3x3 matrix that encodes a 3D transformation, how does singular value decomposition separate the matrix into scale and rotation components?
8. There are 8 functions plotted below. Neatly cross out the ones that are not part of the cubic B-spline basis set. Number the remaining functions to show the order that they go together to form the B-spline “hump” function.

For those that are NOT B-spline basis functions write a single short sentence that explains why they could not be. Your reason should be simple. Note: “It isn't what I have in my notes,” “it won't fit,” “it doesn't solve the equations,” or other generic answers will not be accepted.

<table>
<thead>
<tr>
<th>Letter</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
9. You are working on the shader for computer animated character that is supposed to look as realistic as possible. You start with a basic Phong model and unfortunately no matter how you set Ks and Kd, the results look more like a figure made out of plastic or painted plaster than a real human. Name a missing phenomenon that might be a significant part of the explanation as to why your results don't look like real human skin?  

5 points

10. The following are the response curves for the cones in the human eye. Which type of cone is most sensitive to blue light?  

2 points

11. Give two examples of lighting phenomena that cannot be computed using a strictly local illumination model.  

4 points
12. A friend of yours tells you that they have a method for computing an analytical inverse to any kinematics equation. Give two succinct reasons why this claim is very unlikely to be true. (Claiming you have no smart friends is not a valid answer.) 8 points

13. Consider the diagram below. A location has been marked on the surface. Indicate a viewer position such that a viewer looking at the surface from that position would see a specular highlight on the surface at the marked location. 3 points

14. Given a rotation matrix, what is a simple method you could use to determine its axis of rotation? 4 points
15. Explain the distinction between “motion capture” and “performance capture”?  

5 points

16. Consider this diagram showing a four-joint arm in 2D where each joint is a simple pin joint and the base is fixed in place.

If we are solving an IK problem to place the tip of the arm (the black dot) at a particular location, what is the size of the Jacobian matrix we would be working with?  

3 points

Draw a configuration of the arm showing a target that is out of reach with the arm at the closest possible solution. Draw the vector that would correspond to the non-zero singular value of the Jacobian in this configuration.  

8 points
17. Name two types of useful image effects that would require shooting more than one ray per pixel in a ray-tracer. 

4 points

18. On the diagram below, draw the springs that could be added to provide some resistance to bending.

4 points

19. Given a point, A, and an normal, N, write out an implicit equation for the plane in 3D that contains A and that is normal to N.

4 points
20. Given a point, A, and a distance, r, write out an implicit equation for the sphere in 3D that is centered at A with radius r. 

4 points

21. The diagram below is the control polygon for a Bezier curve segment. Draw the curve and show how de Casteljau’s algorithm can be used to subdivide the curve into two halves. Make sure your drawing is geometrically reasonable and shows correct curve tangents for the beginning, middle, and end of each segment.

6 points
Given a plane and a sphere:

Plane: \( \hat{n} \cdot x + f = 0 \)

Sphere: \( ||c - x|| - r = 0 \)

Write out pseudo code that will compute and return the point on the plane that is closest to the center of the sphere unless that point is not inside the sphere in which case it will return the point on the sphere that is closest to the plane.

*Your answer must be neat and clear. No points will be awarded for imprecise or messy answers. It's sloppy or hard to read then it will get no credit.*