Final Exam

CS 184: Foundations of Computer Graphics
Fall 2013
Prof. James O'Brien

Student Name:

Class Account Username:

Instructions: ***Read them carefully!***

The exam begins at 7:10pm and ends at 10:00pm. You must turn your exam in when time is announced or risk not having it accepted.

Make sure you fill in your name and the above information, and that you sign below. Anonymous tests will not be graded.

Write legibly*. If the person grading the test cannot read something, he/she will simply assume that you meant the illegible portion as a note to yourself and they will ignore it. If you lose points because part of your answer could not be read, you will not be given the opportunity to explain what it says.*

Be clear and concise*. The answers to most questions should be short. If you find yourself writing an excessively long response, you may want to think more carefully about the question. Long rambling answers generally get fewer points that short ones do because there are more opportunities to mark something wrong.*

You may use two pages of notes while taking the exam. You may not ask questions of other students, look at another student's exam, use a textbook, use a phone or calculator, or seek any other form of assistance. In summary: do not cheat. Persons caught cheating will be subject to disciplinary action.

Do not ask questions during the exam. *Most questions are unnecessary and they disturb other students. Figuring out what the exam question is asking is part of the test. If you think you have to make some unusual assumption to answer a problem, note what that assumption is on the test.*

I have read these instructions, I understand them, and I will follow them.

Your Signature: _____

Date: _____

Student ID: _____

Total Points: 250 + 12 *You Scored:* _____ + *Extra* _____

1. Please fill in each of the blanks with an appropriate answer. *2 points each blank, 48 total*

The Euler integration scheme tends to cause simulations to "blow up." The implicit version of this scheme, known as _____, is much more stable but has a tendency to damp motions artificially.

_____ integration schemes make use of the accelerations at the *beginning* of each simulation timestep.

_____ motion capture systems use soft, retro-reflective markers attached to the subject.

If the singular value decomposition of a matrix is $\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}}$, then the psuedo-inverse of the matrix is given by $\mathbf{A}^{-\mathrm{P}} =$_____.

The gradient vector of an implicit surface generally can be used to computed the surface _____ vector.

When representing _____ in 3D using homogenized coordinates, the fourth coordinate (i.e. "w") will be *zero*.

_____ approximates global illumination effects by making diffuse shading proportional to the un-occluded area over a surface.

Radiosity is measured in units of _____.

Catmull-Clark subdivision surfaces are a generalization of uniform, cubic, tensor-product _____ surfaces.

A Bézier curve is always enclosed by the _____ of its control points.

In Catmull-Clark subdivision, the number of new polygons that are not quads introduced by the second round of subdivision will be _____ .

The _____ in the human eye are used in dimly lit situations.

The _____ of an orthonormal matrix is equal to its *inverse*.

_____ encode 3D rotations as 3D points inside a ball of π radius.

A so-called directional light source is the limit case of a _____ light that is infinitely far away from the scene it illuminates.

An orthographic projection is a special case of perspective projection where the _____ is infinitely far away from the scene.

A texture mapping method called _____ is used to create the appearance of small-to-mid scale surface geometry, but it does not actually change the object's shape.

NURBS are non-uniform _____ B-Splines that use homogeneous coordinates for control points.

_____ are the dimensionless units used to measure solid angles.

Finding the intersection of a ray with a sphere requires solving a _____ equation.

Finding the intersection of a ray with a plane requires solving a _____ system of equations.

A rigid body will have a constant rotational _____ unless some external force acts on it.

The dynamic range of the human eye is much _____ than the dynamic range of a typical camera.

If a spring with length l has stiffness coefficient k, then a pair of springs in serial with length l/2 should have stiffness _____ if they are to replicate the behavior of the original spring.

2. Answer the following questions with True (T) or False (F) *2 points each, 100 total*

_____ Kinematic light transport can be modeled reasonably well using a collection of particles attached by springs.

_____ The Form Factor matrix of a valid Radiosity system will have eigenvalues all less than one.

_____ Shinny plastic surfaces typically have bright white specularities.

_____ Radiance remains constant along spiral arcs in free space.

_____ The explicit representation of a given geometric entity is unique.

_____ The rods in the human eye have a spectral response function that peaks between the *short* and *medium* cones.

_____ Under linear perspective projection, squares always appear as quadrilaterals unless the projection is degenerate.

_____ Under linear perspective projection, triangles always will appear to have at least two angles greater than 90 degrees.

_____ Under orthographic projection, all sets of parallel lines will remain parallel.

_____ Quaternions represent rotations as points in 4D space on the surface of a hypersphere.

_____ Any set three of non-intersecting polygons can be sorted in front-to-back order.

_____ LCD monitors could be designed to use other colors besides red, green, and blue.

_____ Shining an ultraviolet light on scorpions induces a chemical response that causes them to glow green and eventually die.

_____ The force exerted by a Hookean spring with non-zero rest length is given by a function that is nonlinear in terms of the endpoint locations.

_____ Cubic B-splines curves will be C^2 across segment boundaries.

_____ Light fields describe the light traveling through some volume of space.

_____ In a kinematic skeleton, every parent body must have exactly one outboard joint.

_____ A rotation matrix always has determinant of +1.

_____ Pasteurized coordinates light vectors of reflection quaternions.

_____ Ambient occlusion tends to enhance the appearance of surface detail.

_____ The sky is blue because water vapor scatters light in the short part of the visible spectrum.

_____ In some men red-green color blindness is caused by a mutation in the coding for the cones.

_____ A radiosity solver produces a view-independent solution.

_____ Springs can be used to implement weak bending resistance for cloth sheets.

_____ In a rectilinear spring mesh, adding diagonal springs will help to limit shearing movement.

_____ Motion graphs used to animate human figures typically contain many cycles.

_____ Non-planar inverse kinematics problems will always have simple closed-from solutions.

_____ A ball joint represented with an exponential map has three degrees of freedom.

_____ C^1 continuity does not always imply G^1 continuity

_____ G^1 continuity does not always imply C^1 continuity

_____ The Bezier basis functions are affine invariant.

_____ The fully explicit version of Euler's method (a.k.a. forward Euler) is fairly unstable.

_____ The human eye is uniformly sensitive to all frequencies of visible light.

_____ Perspective transformations can distort straight lines into parabola.

_____ Some motion capture systems use magnetic fields to determine the location and orientation of tracker objects.

_____ Particle systems simulate objects such as waterfalls by modeling the detailed interactions between individual molecules of water.

_____ The result of applying subdivision to a cubic curve is two eliptic curves.

_____ Raytracing can be accelerated using BSP-Trees.

_____ Raytracing can be accelerated using AABB-Trees.

_____ The rods in the human eye only sense white light.

_____ In a bounding-box tree, the bounding-box stored at a root note will encompass the objects stored at its children's nodes.

_____ The Hermite basis functions have radial-axis support.

_____ Bump-mapping will change an object's volume.

_____ Rational polynomial basis functions can be used to build perfect circles.

_____ Z-Buffers can be used for hidden surface removal.

_____ Z-Buffers can be used to generate shadow maps.

_____ Smoothed particle hydrodynamics (SPH) is a method for modeling fluid flow.

_____ An incompressible fluid should be divergence-free.

_____ A moving object's moments of inertia must remain constant.

_____ Calculations with floating-point numbers produce exact results.

3. Imagine that you have a RGB monitor where the wires have been swapped so that the red, green, and blue outputs from the computer have been respectively attached to the green, blue, and red inputs on the monitor. When one attempts to display the following colors, what colors will actually appear on the screen? *8 points*

Red _____

Green _____

Blue _____

Cyan _____

Magenta _____

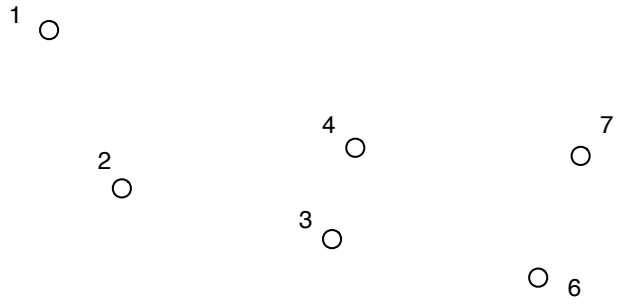
Yellow _____

Black _____

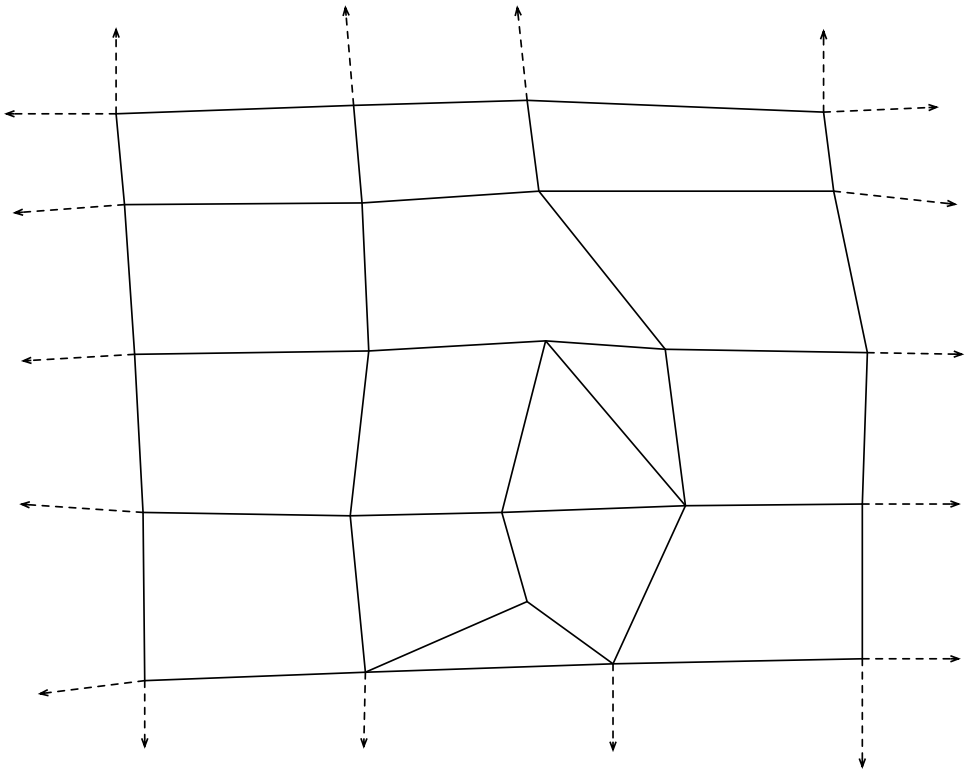
White _____

4. If a surface in 3D is defined implicitly by the function $f(\mathbf{x}) = 0$ (negative inside), write out the equation you would use to compute the surface's outward pointing normal at some point. *4 points*

5. The diagram below shows control points for a curve made by joining two cubic Bezier segments. However control point #5 has been removed. Indicate a location where #5 may be placed to achieve C^1 continuity and <u>draw the curve that would result</u>. Also draw a line where #5 may be placed to achieve G^1 continuity. *Make sure your diagram is clear and geometrically reasonable.* *6 points*



6. Here is a piece of mesh. Draw the result of applying one iteration of Catmull-Clark subdivision. Then circle all vertices (both original and the new ones you added) that are extraordinary. *Note: I am only interested in the topology of your answer.* *8 points*



7. Name a phenomenon that can be modeled easily using ray-tracing but that cannot be modeled with a basic radiosity algorithm. Give an example. *3 points*

8. Briefly describe a useful method for *averaging* two rotations. *6 points*

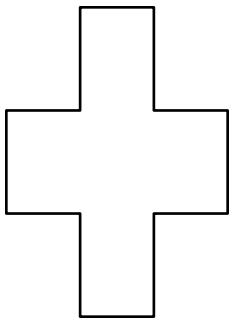
9. Below are two 4x4 homogenized transformation matrices. What does the first one do? How does the effect produced by the second one differ from that produced by the first? *4 points*

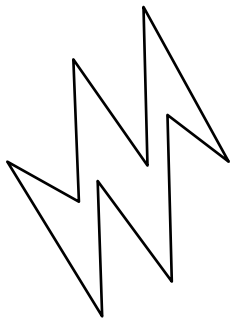
$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & -16 & 0 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

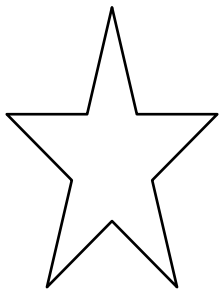
The first one will: _____

The second one will: _____

10. Draw the convex hulls for each of the individual shapes shown below. *6 points*



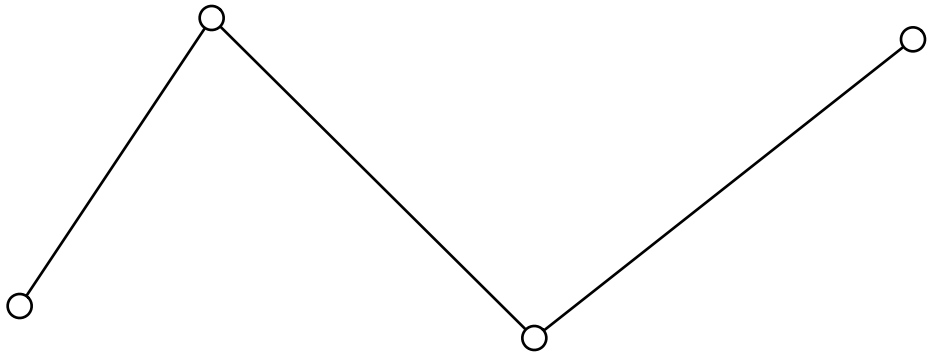




11. Write out a parametric equation for a unit radius sphere centered at the origin. *6 points*

12. Write out an implicit equation for a plane in 3D passing through the point [4,2,3] with normal given by [0,0,1]. *3 points*

13. The diagram below is the control polygon for a Bezier curve segment. Draw the curve and show how de Casteljau's algorithm can be used to subdivide the curve into two equal halves. *Make sure your drawing is geometrically reasonable and shows correct curve tangents for the the beginning, middle, and end of each segment.* *5 points*

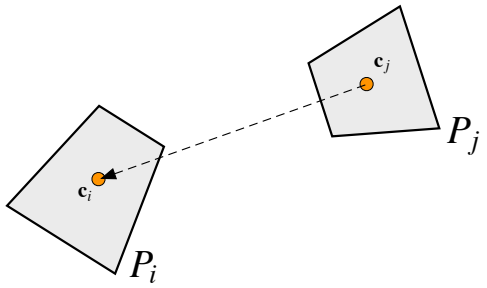


14. In the space below draw a diagram illustrating the formation of a tight *reflection caustic*. Your diagram should be clear and concise, and clearly show the light source(s), the reflecting surface(s), the surface where the caustic forms, and the paths of representative rays. A 2D diagram is suggested. *4 points*

15. Given four points, A, B, C, and D write out a bilinear parametric equation in two variables, *u and v,* for the quadrilateral formed by these four points. *6 points*

16. Consider the following equation and diagram:

$$H_i = E_i + \rho_i \sum_j H_j \int_{P_j} \delta_{ij} \frac{\cos(\theta_i)\cos(\theta_j)}{2\pi||\mathbf{c}_i - \mathbf{x}||^2} \mathrm{d}\mathbf{x}$$



Explain what effects each of the following is responsible for. *14 points*

H_i _____

E_i _____

ρ_i _____

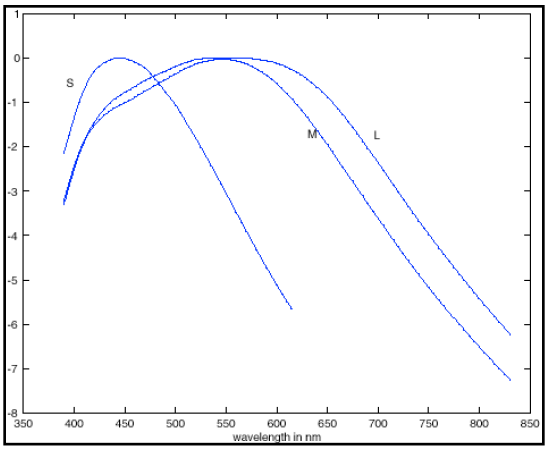
The integral term _____

$\cos(\theta_i)$ _____

$\cos(\theta_j)$ _____

δ_{ij} _____

17. The following are the response curves for the cones in the human eye. Which type of cone is *most* sensitive to *blue* light? *2 point*



18. Consider the diagram below. Given the indicated location of the light and the viewer, mark on the line where a specular highlight would occur. Draw the light path and the normal of the surface. Indicate where any lengths of angles are the same as other lengths or angles in the diagram. *3 points*

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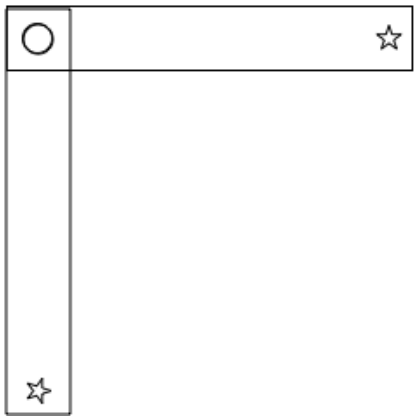
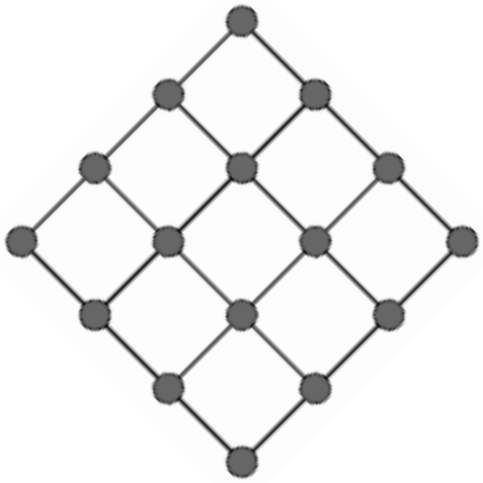
19. When rendering a scene with a photon mapping method, what part of the solution must be recomputed when the viewer moves? 2 points

20. Below is a diagram showing a bar that has been rotated 90 degrees about the point indicated with a circle. If intermediate positions were generated by linearly interpolating the transformation matrices, how would the point indicated by the star move? Give your answer by drawing the path of the star. *4 points*



21. On the diagram below, draw the springs that should be added to provide some resistance to in-plane shearing. *4 points*



22. I define a curve as $\mathbf{c}(u) = \sum_i \mathbf{p}_i \phi_i(u)$, $u \in [0..1]$ **for some set of basis functions** $\phi_i(u)$ **and I insist that no mater what values are used for the control points** $\mathbf{p}_i$**, the resulting curve is always in the convex hull of the control points. Specify exactly and clearly what mathematical conditions must be true for my assertion to hold?** *4 points*

EXTRA CREDIT *12 points*

Given a two planes where the first is defined implicitly and the second parametrically:

Plane 1: $\mathbf{x} \cdot \mathbf{n}_1 - \mathbf{a}_1 \cdot \mathbf{n}_1 = 0$

Plane 2: $\mathbf{x}_2(u, v) = \mathbf{a}_2 + u\mathbf{b}_2 + v\mathbf{c}_2$

Write out an explicit parametric equation that produces the line where the two planes intersect. Also indicate when this line is undefined.

Your answer must be neat and clear. No points will be awarded for imprecise or messy answers. Your answer should be in the form of a simple explicit equation that you have drawn a box around. Do not attempt this question until you have completed the rest of the exam! Generally there will be no partial credit for this question.