Problem #1
(A)

Here we are looking for exactly two of something, so use the general inclusion exclusion formula:

\[
\sum_{i=0}^{\lfloor k/2 \rfloor} (2+i)C(i) \times (k-2-i)C(2+i) \times 5^{k-2-2i}
\]

(B)

In this problem count the case AAA seperately... note the rest of the string must not contain AA

Then add on the case with AA and AA delimited by non A characters

Problem #2
The checker should pick a random vector \( v \), then for the supposed inverse, \( B \), compute \( A(Bv) \) and compare this with \( v \). If they are different, then \( B \) is not \( A \) inverse, otherwise there is less than 50% chance it is not the inverse. This must be repeated at least \( n \) times with linerly independent \( v \)'s to be sure that \( B \) is the inverse. Each repetition requires two matrix, vector multiplies, or \( O(n^2) \) operations.

Problem #3
When appending \( T \) after \( S \), consider the changes to the first level of nodes above the actual string. Only the last above \( S \), and the first above \( T \) will change. This is a constant number of changes which will be propogated up the fingerprint tree. So we have a constant number of changes at each level, and the number of levels is bounded by \( \log_2 n \), and therefore \( O(\log n) \) new nodes, and \( O(\log n) \) time to append.

Problem #4
(A) No. Once one of the good processors has tally \( \geq G \) then on the next round all the good processors will have tally \( \geq G \), and have all set their votes permanently.

(B) A processor can halt one round after it has set its vote permanently.

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