

CS174 Midterm Sept. 25, 2003

1. Let A and B be events in the same sample space, such that $Pr(A) = 2/5$, $Pr(B) = 3/10$ and $Pr(A|B) = 2/3$. What is $Pr(B|A)$?
2. In a permutation $\pi = (\pi(1), \pi(2), \dots, \pi(n))$, index i is called a cumulative maximum if $\pi(i) = \max(\pi(1), \pi(2), \dots, \pi(i))$. What is the expected number of cumulative maxima in a random permutation of $\{1, 2, \dots, n\}$?
3. An airline knows that on average five percent of the people making reservations on a certain flight will not show up. Consequently, their policy is to sell 52 tickets for a flight that can only hold 50 passengers. Give a formula involving binomial coefficients (but not involving the summation symbol) for the probability that there will be a seat available for every passenger that shows up.
4. Let S be a set of n elements. At the first stage each element in S is independently removed with probability p . Those elements not removed constitute the set S_1 . If S_1 is not the empty set then each of its elements is independently removed with probability p , with the remaining elements constituting the set S_2 , and so on. Let N be the least k such that S_k is empty. Give a formula for the probability that $N = r$. Hint: N can be expressed as the maximum of a set of independent geometric random variables.
5. A rat is trapped in a maze. Initially it has to choose one of two directions. If it goes to the right it will wander around in the maze for three minutes and will then return to its initial position. If it goes to the left then with probability $1/3$ it will depart the maze after two minutes of traveling, and with probability $2/3$ it will return to its initial position after five minutes of traveling. Assuming that the rat is at all times equally likely to go to the left or to the right, give the expectation of the number of minutes that it will be trapped in the maze.
6. Suppose that X is a random variable with mean 10 and variance 15. What can we say about $Pr(5 \leq X \leq 15)$?