

**CS 172, Spring 1999
Midterm Exam 2
Manual Blum**

- This is a CLOSED BOOK examination.
- Calculators ARE permitted.
- Do all your work on the pages of this examination.
- For most problems, you may answer YES, NO, "I think so", "I think not", or "I don't know".
- Give reasons -- carefully written correct reasons -- for your YES/NO answers, and to the extent you can for your "I think such and such" answers.
- This examination has 7 problems, worth a total of 97 points.

Problem #1

(1 pt)

a) The product of binary numbers 1011 and 1101 is

1011* 1101 = _____.

(10 pts: 1 pt each; double this if all are correct)

b) In spaces to the right, write YES or NO (or "I think so" or "I think not" or "I don't know").

The length (in bits) of the product of an m-bit number and an n-bit number, for positive integers m and n, is:

- $O(mn)$ bits _____
- $O(m+n)$ bits _____
- $m+n+O(1)$ bits _____
- $\text{MAX}\{m,n\}$ bits _____
- $\text{min}\{m,n\}$ bits _____

Define the Computational Decision Problem ZMP as follows:

DECISION PROBLEM ZMP (Zeroes of a Multivariate Polynomial)

INSTANCE:

- A multivariate polynomial* with interger coefficients.
- All coefficients given in binary.
- All degrees given in unary or binary, depending.

QUESTION: Does the given polynomial have a root over $\{0,1\}$?

EXAMPLE: The same f as it appears with unary and binary degress

Unary degrees: $f(x,y,z) = (1+x)^{11} + (y^{11111}) \cdot (1+z^{111111}) - 3$

Binary degrees: $f(x,y,z) = (1+x)^2 + (y^4) \cdot (1+z^6) - 3$

Problem #2

(5 pts)

Does the above $f(x,y,z)$ have a root in $\{0,1\}$?

Note: the answer to this question is independent of whether the degrees are given in unary or binary.)

If not, why not?

If yes, give a root:

x=_____

y=_____

z=_____

(Careful! In our problem, the above variables are only permitted to be 0 or 1!)

*DEFINITION: A MULTIVARIATE POLYNOMIAL $f(x_1,x_2,\dots,x_k)$ is defined as follows:

1. Any integer is a polynomial. Ex: 37
2. Any variable from a given finite set of variables, eg $\{x_1, \dots, x_k, x, y, z\}$ is a polynomial. Ex: y
3. A sum or product of 2 polynomials, placed inside parentheses, is a polynomial. Ex: $(x+y)$, and $(37*x)$ (or its equivalent $(37x)$)
4. A polynomial placed inside parentheses and raised to an INTEGER power, is a polynomial.

Parentheses may be left out if the meaning of the polynomial remains clearly unchanged.

Ex: $f(x,y) = ((37*x)^3 + x + 25y)^5$

Problem #3

(20 pts)

Is ZMP in NP when degrees are in unary?

Problem #4

(20 pts)

Is ZMP in NP when degrees are in binary?

Problem #5

(20 pts)

Is ZMP NP-hard when degrees are in unary?

(Recall: "NP-hard" is the same as "Complete for NP". In particular, a decision problem is NP_complete iff it is in NP and it is NP-hard.)

Problem #6

(20 pts)

Is ZMP NP-hard when degrees are in binary?

Problem #7

(1 pts)

The CHEESE problem: Check your answers to problems 1-6 above. Then hand in your exam and exit BECHTEL, this floor, to find and solve the (experimental) cheese problem.

**Posted by HKN (Electrical Engineering and Computer Science Honor Society)
University of California at Berkeley
If you have any questions about these online exams
please contact examfile@hkn.eecs.berkeley.edu.**