Each of the following questions counts equally. Try to keep your answers succinct.

**Pumping Lemma:**

If \( L \) is regular then

\[
(\exists n)(\forall z \in L, |z| \geq n)(\exists uvw \text{ such that } z = uvw \text{ and } |uv| \leq n \text{ and } |v| \geq 1)(\forall i): uv^iw \in L.
\]

1. Let language \( L \) be given by the regular expression \( 10^*1 \).
   (a) Construct a DFA accepting \( L \).
   (b) Construct a DFA accepting \( \overline{L} \).
   (c) Construct a regular expression for \( \overline{L} \). If your expression is complicated, you should be able to give a succinct overview in English to convince me that your expression is correct.

2. We wish to prove that \( L = \{0^i1^j : \gcd(i, j) = 1\} \) is not regular. Recall that \( \gcd(i, j) = 1 \) if \( i \) and \( j \) have no factors in common. So, \( 0^{10}1^3 \in L, 0^{5}1^5 \not\in L \) and \( 0^{6}1^{10} \not\in L \). Here are three “proofs” that \( L \) is not regular; one correct. Identify the correct proof with a \( \star \), and succinctly explain what is wrong with the other two proofs. (Hint: The incorrect proofs use the pumping lemma wrong — nothing is wrong with the algebra.)

   (a) It suffices to show that \( \overline{L} \) is not regular. Fix \( n \) in the pumping lemma. If \( z = 0^n1^p \in \overline{L} \) for prime \( p > n + 1 \) and let \( z = uvw \) as in the lemma. No matter what \( uvw, uv = 0^n1^p \) for some \( 1 < l < p \) and \( \gcd(l, p) = 1 \).

   So \( uv \not\in L \) and \( \overline{L} \) is not regular.

   (b) Fix \( n \) in the pumping lemma. Note that consecutive numbers above 1 cannot have a common factor. So the string \( z = 0^{2n+1}1^{2n} \in L \). Choose \( z = uvw \) as in the lemma, where \( v = 0^l \) for \( l \) odd. \( uv = 0^{2n+1}1^{2n} \not\in L \) since both \( 2n + 1 - l \) and \( 2n \) are divisible by 2. Hence \( L \) is not regular.

   (c) Fix \( n \) in the pumping lemma. Without loss of generality, \( n \geq 2 \). Choose \( z = 0^n1^n \not\in L \) and fix \( z = uvw \). Now \( v \) must be of the form \( 0^l, 1 \leq l \leq n \). Then \( uv^{n+1}w = 0^{n+ln}1^n \not\in L \) since both \( n + ln \) and \( n \) are divisible by \( n \). Hence, \( L \) is not regular.