Midterm 1

Name:

This test has three questions and four numbered pages. Answer on the question paper.

You may use any result that was proved in lectures without giving the proof, as long as you state the result clearly. (It is not necessary to state the theorem number.) For full marks, you must justify your answers.
Question 1 (25 marks)

For any \( k > 0 \), let \( L_k \) be the language of binary strings that do not contain \( k \) consecutive 1s. For each \( k \), define a DFA for \( L_k \) and prove that it is minimal.
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Question 2 (25 marks)

Let $L \subseteq \{0,1\}^*$ be an infinite language. Show that $L$ is decidable if, and only if, there is an enumerator for $L$ that outputs the strings in $L$ in lexicographic order.

(Recall that the lexicographic order on strings puts $u$ before $v$ if, and only if, either $|u| < |v|$ or $|u| = |v|$ and $u$ comes alphabetically before $v$, i.e., there is some $j \leq |u|$ such that $u_i = v_i$ for $1 \leq i < j$ and $u_j < v_j$.)
Consider an NFA $N = (Q, \Sigma, \Delta, q_0, A)$ that has no $\epsilon$-transitions (i.e., $\Delta(q, \epsilon) = \emptyset$ for every $q \in Q$). Recall that $N$ accepts its input if there is some possible sequence of transitions that leads to an accepting state.

We define a for-all finite automaton (\forall FA) identically to an NFA with no $\epsilon$-transitions, except that we say that a \forall FA $F$ accepts input $w$ if, and only if, every possible sequence of transitions when reading $w$ leads to an accepting state.

a) Show that the language accepted by a \forall FA is regular.

b) Is every regular language accepted by some \forall FA?