CS 172: Computability and Complexity, Spring 2010 **S. A. Seshia & O. Etesami**

Midterm 2

April 14, 2010

YOUR NAME:

Instructions:

This exam is *open-book, open-notes*. Please turn off and put away electronic devices such as cell phones, laptops, etc.

You have a total of 75 minutes. There are 4 questions worth a total of 100 points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it for a while and try another one.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page. *You can use without proof any result proved in class, in Sipser's book, or on homeworks, but clearly state the result you are using.*

Do not turn this page until the instructor tells you to do so!

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

Problem 1: [True or False, with justification] (30 points)

For each of the following four questions, state TRUE or FALSE. Justify your answer with a short proof or simple counterexample.

(a) Let A be NP-complete and B be NP-hard. Then $B \leq_P A$.

(b) The set of all Turing-recognizable languages is countably infinite.

(c) If L_1 and L_2 are Turing-recognizable, then $L_2 \setminus L_1$ is also Turing-recognizable.

Problem 2: (25 points)

Let $B = \{\langle Q, \Sigma, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, n, w\rangle \mid$ there are at least n transition functions δ such that the Turing machine $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ accepts the string $w\}$.
Prove that B is undecidable.

Problem 3: (20 points)

A *Random-Access Turing machine* (RA-TM) is similar to the standard, single-tape, deterministic TM except for its transition function. On a transition, a RA-TM can move its head a finite, but arbitrary distance from its current location. Formally, the transition function is

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times (\{L, R\} \times \mathbb{N})$$

For example, $\delta(q_i, b) = (q_j, c, (L, n))$ will cause the RA-TM's head to move n places to the left (stopping at the left-most cell if the jump would cause the head to move off the tape).

Prove that every RA-TM has an equivalent standard, single-tape, deterministic TM.

Problem 4: (25 points)

Given a graph $G = (V, E)$, an *independent set* is a set of vertices V' such that $V' \subseteq V$ and no two vertices in V' are connected by an edge in E.

The problem of determining the *maximum-size independent set* in a graph is known to be NP-complete. Here is the decision version of that problem:
$IS = \{\langle G, k\rangle | G$ is an undirected graph with an independent set of size $k\}$

In this question, we investigate two related problems. (Be sure to turn the page for the second part!)

(a) Suppose that G is a tree. Consider the following special case of the *INDSET* problem:
$IST = \{\langle G, k\rangle | G$ is an undirected tree with an independent set of size $k\}$

Show that $IST \in$ P.

(b) Now consider the following variant of the problem of finding the independent set in a general undirected graph.

$$\text{SpecialIS} = \{\langle G, k, v\rangle \mid G \text{ is an undirected graph with an independent set of size } k \text{ that includes vertex } v\}$$

Show that SpecialIS is NP-complete.