CS172, Spring 2001
Final Exam

This exam is open-book. There are 6 full-length problems, 3 short-proof questions, and an extra-credit question. You have 170 minutes, although the exam is designed to take slightly less than that. The number of points assigned to each problem reflects the number of minutes expected to be spent on it, and there's a total of 160 points. We are not looking for rigidly formal construction proofs unless otherwise specified, but you should give sufficient detail to demonstrate that you can make the necessary construction based on constructions presented in class and/or in the textbook. Good luck!

1. (20 pts) Consider the following simple recipe for passing CS172:

   (i) Prior to the exam, you must study at least once.

   (ii) You must sleep and/or party at least once after any two consecutive studying sessions that precede the exam.

   (iii) You must sleep right before the exam.

   (iv) You must take the exam exactly once.

   (v) You **must** party right after the exam, and then you're free to do whatever you want.

   (vi) Any violation of rules 1-5 does not result in a passing grade.

   Consider the language of sequences of actions, defined over the alphabet $\{S, L, P, E\}$ (Study, sLeep, Party, take-Exam) of possible actions, that lead to a passing grade. To demonstrate just how simple this language is, show that it is regular by providing a **DFA**.

   NOTE: To simplify notation, you may assume that any arrow not explicitly shown in your diagram leads to an implicit **rejecting** state (from which there is no way out).

2. (15 pts) Give a CFG for the following language, or prove that it is not context-free:

$B = \{w\#x^R\#y | w, x, y \in \{0,1\}^*;\ x \text{ is a substring of } w \text{ and } y \text{ is a substring of } x\}$

3. a. (10 pts) $2SAT$ is defined analogously to $3SAT$ except that each clause has exactly 2 literals. Explain the fallacy in the following reduction to prove that $2SAT$ is NP-complete:

Proof: Clearly $2SAT$ is in NP. Given a $2SAT$ formula $\phi(x_1, \ldots, x_n) = c_1 \wedge c_2 \wedge \ldots \wedge c_m$, we transform it to a $3SAT$ formula as follows. For each clause $c_i = (l_i \vee m_i)$, we introduce a new variable y_i and we replace clause c_i by the conjunction of two new clauses $(l_i \vee m_i \vee y_i) \wedge (l_i \vee m_i \vee \bar{y}_i)$. Let $\psi(x_1, \ldots, x_n, y_1, \ldots, y_m)$ denote the formula that results. $\psi(x_1, \ldots, x_n, y_1, \ldots, y_m)$ is a $3SAT$ formula that is satisfiable if and only if $\phi(x_1, \ldots, x_n)$ is satisfiable. Therefore $2SAT$ is NP-complete.

b. (25 pts) Consider the problem of solving SAT when we're given a "hint" for the solution – an assignment which satisfies all but 1 clause of the formula. We formalize this as the language $NEAR\text{-}SAT$:

$\{\langle \phi, x_1, \ldots, x_n \rangle | \phi$ is a CNF formula of n variables; assignment $x_1, \ldots, x_n$ satisfies all but 1 clause$\}$.

Prove that, despite the apparent "hint", $NEAR\text{-}SAT$ is still NP-complete. Hint for solution: you should use SAT for your reduction (unlike the hint in $NEAR\text{-}SAT$, this one should be helpful).

$NEAR\text{-}SAT \in NP$:

REDUCTION:

JUSTIFICATION:

4. (25 pts) A directed graph $G(V, E)$ with $V = \{0,1\}^n$ is implicitly given as follows: the edges of G are specified via a polynomial time Turing Machine E, which on input $s, t \in \{0,1\}^n$ accepts iff $(s,t) \in E$. Define the language $PATH = \{\langle E, s, t\rangle | \text{there's a path from } s \text{ to } t \text{ in the graph represented by } E\}$. Prove that $PATH \in PSPACE$.

   Hints: Start by proving $PATH \in NPSPACE$. Note that counting up to k requires only $\log k$ bits.

5. (20 pts) Let $A = \{(x, y) : x, y \in \{0,1\}^* \text{ and } |x| = |y|\}$. Let $B \subseteq A$ be a language such that $B \in BPP$. Let $C = \{x : \exists y \text{ s.t. } (x, y) \in B\}$. Prove that $C \in IP$ (use the definition of IP directly; you may **not** use Shamir's $IP = PSPACE$ result).

6. (24 pts) **Short proofs**. For each of the statements below, mark the statement as either True or False, and give a **brief** (no more than 2-3 sentences) justification of your answer. Each is worth 8 points.

   Notation: $A \backslash B$ is "set subtraction", equivalent to $A \cap \overline{B}$.

   a. If A is Turing-recognizable, and $A \leq_m \overline{A}$, then A is decidable.

      TRUE FALSE

   b. If A and B are Turing-recognizable, then $A \backslash B$ is decidable.

      TRUE FALSE

   e. $P \subsetneq EXPSPACE$.

      TRUE FALSE

7. We say that a Turing Machine M loops on input w if it repeats some configuration c. Let $L_{loop} = \{\langle M, w\rangle | M \text{ loops on } w\}$.

   a. (10 points) Show that L_{loop} is recursively enumerable.

   b. (11 points) Use the recursion theorem to show that L_{loop} is not decidable.

8. (Extra-credit; 15 pts) Do **not** attempt the question below until you have finished the rest of this exam.

   Suppose next week someone announces and provides a conclusive proof that $P \neq NP$. In a paragraph, explain what impact you believe such a result would have on the field of computer science and on society at large.