Problem #1

Closure properties of languages associated with Turing machines.
(a) Prove that the union of two Turing-recognizable languages is Turing recognizable.
(b) Prove that the union of two decidable languages is decidable.

Problem #2

Show that the class of context-free languages is closed under *. (Hint: It is probably easiest to do this with a grammar, where a fairly simple construction suffices, but it is doable with PDA's if you prefer).

Problem #3

(a) Draw an NFA that recognizes the languages:

\[ A = \{w \in \{a,b,c\}^* | w \text{ contains at least two b's or at least one c} \} \]

(b) Give a regular expression that describes this language.

Problem #4

Design a PDA for

\[ L = \{(0^i)(1^j) | i \neq j \text{ and } i, j \geq 0 \} \]

A high-level English description will get you partial credit, and a diagram will receive full credit.

Problem #5

Let \( \sigma = \{0,1,...,9\} \). Let

\[ L = \{ s | \text{ M is a DFA and M does not accept any string containing 555 as a substring} \} \]

Show that L is decidable. (Hint: Use the fact that it is possible to construct a DFA that recognizes the regular language \( \sigma^*555\sigma^* \). Also use the fact that regular languages are closed under intersection.).

Problem #6

Let A and B be Turing-recognizable languages. Let \( (A \cap B) \) and \( (A \cup B) \) be decidable. Show that A and B are decidable. (Hint: Use a Venn diagram and analyze the decidability of various regions of the diagram).

Problem #7

Consider the problem of testing whether a two-tape Turing machine ever writes a nonblank symbol on its second tape. Formulate this problem as a language. Show that this language is undecidable. (Hint: Use a reduction from \( (A)tm \). The basic idea is to construct a two-tape machine that simulates a Turing machine M on string w.).