CS 172 — Fall 1997	**Prelim 1**
Computability and Complexity	**October 1, 1997**

Problem 1. (75 points) Consider the regular expression $R = (\mathtt{0} \cup \mathtt{10} \cup \mathtt{100})^*$.

a. Draw a nondeterministic finite automaton N such that $L(N) = L(R)$. (You may take shortcuts that omit ε transitions.)

b. Use the subset construction to draw a deterministic finite automaton D such that $L(D) = L(N)$. (Label each state of D with the corresponding set of states of N.)

c. Use the minimization algorithm followed by the quotient construction to draw a minimal DFA M such that $L(M) = L(D)$. (Label each state of M with the corresponding equivalence class of states of D.)

d. What is the index of $L(R)$? If the index of $L(R)$ is k, give k words $w_1, \ldots, w_k$ such that no two of the words are $\equiv_{L(R)}$ equivalent. (You need not justify your answers.)

e. Give a regular expression S such that $L(S) = [\mathtt{1}]_{\equiv_{L(R)}}$. (Here $[\mathtt{1}]_{\equiv_{L(R)}}$ stands for the $\equiv_{L(R)}$ equivalence class of the one-letter word $\mathtt{1}$.)

Problem 2. (75 points) For every language $A \subseteq \Sigma^*$, we define the following two languages:

$$double1(A) = \{x_1 x_2 x_3 \ldots x_{2n} \in \Sigma^* \mid n \geq 0 \text{ and } x_1 x_2 x_3 \ldots x_n \in A\}$$
$$double2(A) = \{x_1 x_2 x_3 \ldots x_{2n} \in \Sigma^* \mid n \geq 0 \text{ and } x_1 x_3 x_5 \ldots x_{2n-1} \in A\}$$

a. For $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ and $B = L(\mathtt{0}^*)$, describe in words the two languages $double1(B)$ and $double2(B)$.

b. Which of the following two statements are true and which are false?

S1 If A is a regular language, then $double1(A)$ is also regular.
S2 If A is a regular language, then $double2(A)$ is also regular.

To argue that one of these statements is true, you must define a finite automaton $M' = (Q', \Sigma', \delta', q_0', F')$ that accepts $double(A)$ from a given finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ that accepts A.

To argue that one of these statements is false, you must find a regular language A and prove, using the pumping lemma, that $double(A)$ is not regular.