1. Consider the following problem:
   
   INSTANCE: A binary Turing machine M.
   QUESTION: Does M accept at least 10 strings?

   The above problem can be formulated as the problem of recognizing the language
   
   \[ L_{>=10} = \{ <M> \mid M \text{ accepts at least 10 strings} \} \]

   (Hear <M> denotes the standard encoding of a binary TM.)

   (a) Show that the language \( L_{>=10} \) is recursively enumerable (r.e.).

   (b) Recall that the language \( L_{\text{halt}} \), defined by
       
       \[ L_{\text{halt}} = \{ <M> \mid M \text{ halts on } x \} \]

       is not recursive. By giving a reduction from \( L_{\text{halt}} \) to \( L_{>=10} \), prove that \( L_{>=10} \) is not recursive.
       
       NOTE: You need not show in detail that your reduction can be performed by a TM, but you should show clearly that it maps 'yes'-instances to 'yes'-instances and 'no'-instances to 'no'-instances.

   (c) Is the language
       
       \[ L_{<10} = \{ <M> \mid M \text{ accepts fewer than 10 strings} \} \]

       r.e.? Justify your answer carefully.

2. The Steiner Tree problem, ST is defined as follows:
   
   INSTANCE: An undirected graph \( G = (V, E) \) is a subset \( R \subseteq V \), and a positive integer \( k \).
   QUESTION: Is there a subtree of \( G \) that includes all vertices of \( R \) and contains at most \( k \) edges?

   (This problem arises, for example, when it is required to construct a network linking some collection \( R \) of sites, using some small number \( k \) of existing links (from the set \( E \)) and perhaps some additional sites from \( V \).

   (a) Consider the following graph \( G \):
with $R = \{ 1, 2, 3, 4, 5, 6, 10 \}$ and $k = 8$. Show that this is a 'yes' instance of ST.

(b) Explain briefly why ST belongs to NP.

(c) Prove that ST is MP-complete.

HINT: Try a reduction from the 3-Dimensional Matching Problem, 3DM. The above example should help you.

(d) Does ST remain NP-complete if we restrict attention to instances in which $R = V$ (i.e., all sites are to be connected)? Justify your answer carefully.