CS-172 Quiz 2 September 23, 1994
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Try to keep your answers succinct.

1. (10 points) Let L be regular. As in the homework problem from class, define

$$\mathrm{ALT}(L) = \{a_1 a_3 a_5 \cdots a_{2n-1} : a_1 a_2 a_3 \cdots a_{2n} \in L\}$$

   If L is given by the regular expression $(110)^*$, give a regular expression for $\mathrm{ALT}(L)$.

2. Let L be **any** infinite language that contains all but a finite number of strings.
   (a) (3 points) Give an example of such a language L.
   (b) (7 points) Show that **any** such language, L, is regular.

3. Language L over $\Sigma = \{0, 1\}$ is defined by its complement:

$$\overline{L} = \{(01)^{n^2} : n \geq 0\}$$

   So, typical strings in L include 10, 0101, and 011, but the strings ϵ, 01 and 01010101 are not in L since 0, 1 and 4 are perfect squares.
   (a) (5 points) Show the consequence of the pumping lemma holds for L. I.e., prove that

$$(\exists n)(\forall z \in L \text{ such that } |z| \geq n)(\exists uvw \text{ such that } z = uvw \text{ and } |uv| \leq n \text{ and } |v| \geq 1)(\forall i) : uv^i w \in L$$

   (Remember to consider that i can be 0.)
   (b) (10 points) Despite part (a), prove that L is not regular.