CS172, Fall 1993
Final
David Wolfe

ETA KAPPA NU
Mu Chapter
DEPT. OF ELECTRICAL ENGINEERING
University of California
Berkeley, Calif. 94720

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CS-172
David Wolfe

Cover Sheet to Final

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The last few questions are *all or nothing*. No partial credit will be given on those --blems. So if you have an incomplete solution or a guess, please don't bother to write it down.

Unless otherwise noted, each question is worth 20 points. Try to keep your answers succinct. Feel free to tear off the first 3 sheets, but please leave the rest stapled.

First, a few helpful theorems and definitions. Everything on this page may t be useful.

Lemma: The *Pumping Lemma*:
If L is regular
then $(\exists n)(\forall z \in L$ such that $|z| \geq n)(\exists uvw$ such that $z = uvw$ and $|uv| \leq n$ and $|v| \geq 1)(\forall i)$: $uv^iw \in L$

Lemma: The contrapositive of the *Pumping Lemma*:
If $(\forall n)(\exists z \in L$ such that $|z| \geq n)(\forall uvw$ such that $z = uvw$ and $|uv| \leq n$ and $|v| \geq 1)(\exists i) : uv^iw \notin L$
then L is not regular.

Theorem: *Rice's theorems:* Let $L_{\mathcal{P}}$ be the set of machines with property $\mathcal{P}$. If $\mathcal{P}$ is non-trivial, $L_{\mathcal{P}}$ is undecidable. Further, $L_{\mathcal{P}}$ is r.e. if and only if $\mathcal{P}$ satisfies the following three conditions:

1. If $L \in \mathcal{P}$ and $L \subseteq L'$ for some r.e. L', then $L' \in \mathcal{P}$.
2. If L is an infinite language in $\mathcal{P}$, then there exists a finite subset of L in $\mathcal{P}$.
3. The set of finite languages in $\mathcal{P}$ is *enumerable*.

3-SATISFIABILITY (3SAT)
INSTANCE: A boolean formula, F, which is an AND of clauses where each clause is an OR of 3 literals.

QUESTION: Is F satisfiable?

3-DIMENSIONAL MATCHING (3DM)
INSTANCE: A set $M \subset W \times X \times Y$, where $|W| = |X| = |Y| = q$ are disjoint sets.

QUESTION: Does M contain a matching, $M' \subset M$, such that no two elements of M' agree in any coordinate.

VERTEX COVER (VC)
INSTANCE: A graph G and integer K

QUESTION: Is there a subset of K vertices which cover all the edges?

CLIQUE
INSTANCE: A graph G and integer K

QUESTION: Does the graph contain a clique (comletely connected subgraph) of K vertices?

HAMILTONIAN CIRCUIT (HC)
INSTANCE: A graph G

QUESTION: Is there a cycle through all the vertices of G

PARTITION
INSTANCE: A finite set A and a "size" $s(a) \in Z^+$ for each $a \in A$.

QUESTION: Is there a subset $A' \subset A$ such that

$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)$$

1. (30 points) For the first 3 languages, give examples of strings in L and not in L, and then determine of L is regular. Prove your answer.
   (a) $L = \{0^n : n \text{ is a perfect square}\}$
   (b) $L = \{xwx^R | x, w \in (0+1)^+\}$
   (c) $L = \{xx^R w | x, w \in (0+1)^+\}$
   (d) Consider the following language:

   $$LL^R = \{xy : x \in L \text{ and } y \in L^R\}$$

   We know $L = \{0^n 1^n : n \geq 0\}$ is not regular. What is the language LL^R? Is it regular? Prove your answer.
   (e) If L is regular, is LL^R also regular? Prove your answer.

2. (30 points) Which of the following are r.e.? Give a proof. (Hint: Any reductions can be done from L_u by creating an M' from $\langle M, w\rangle$ which accepts either $\emptyset$ or Σ^* depending on whether $M(w)$ rejects or accepts.)
   (a) $L_{3M} = \{\langle M_1, M_2, M_3\rangle$: At least two of the machines accept the same language.$\}$
   (b) $\overline{L_{3M}}$
   (c) $L = \{\langle M\rangle : M(\epsilon)$ never moves past the $|Q|^{\text{th}}$ tape square$\}$. (Q is the set of states of M.)

3. (a) Show that if L_1 is recursive and L_2 is non-trivial, that $L_1 \alpha L_2$. (A language, L_2, is non-trivial if it's neither $\emptyset$ nor Σ^*. In other words, there is a string $x \in L_2$ and another string $x' \notin L_2$.)
   (b) Show that if P=NP, that any non-trivial language in P is NP-complete.

4. Recall that PSPACE is the set of languages which can be accepted by a deterministic turing machine which, on input w, uses a polynomial in the length of w tape squares. Prove NP $\subseteq$ PSPACE.

5. Show DOMINATING SET is NP-complete:

   INSTANCE: Given a graph $G = (V, E)$ and a positive integer K.

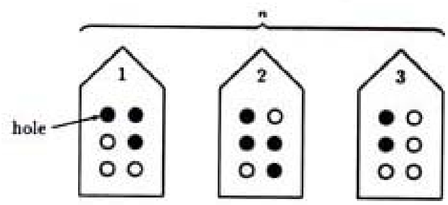
   QUESTION: Is there a subset $V' \subset V$ of fewer than K vertices which covers all vertices of G. (I.e., each vertex is either in V' or adjacent to a vertex in V'.)

   Hint: Reduce from VERTEX COVER.

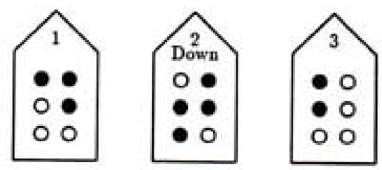
6. Consider the BLOCK THE HOLES problem:

INSTANCE: Integers n and k and a deck of n cards shaped as below with k circles down the left and right sides. Some of the circles are punched out to make holes.

QUESTION: Is there a way to stack the cards, some face up and some face down, so that all the holes are covered (so no light would shine through.)



In the above example, $n = k = 3$ and the $\bullet$'s mark the holes. It is a *yes* instance of BLOCK THE HOLES, since all holes can be blocked by turning cards 1 and 3 face up, and card 2 face down:



Use 3-SAT to show BLOCK THE HOLES is NP-complete. Hint: A card you may *eventually* use is one with holes down one side. The card will serve a role similar to the element representing FALSE in SET-SPLITTING.

7. **(All or nothing)** If L is regular, are the following two languages also always regular? Prove each answer.

   (a) $L_1 = \{xy : x0y \in L \text{ and } |x| = |y|\}$

   (b) $L_2 = \{x0y : xy \in L \text{ and } |x| = |y|\}$

8. **(All or nothing)** Use VERTEX COVER to show SHORTEST COMMON SUBSEQUENCE is NP-complete:

   INSTANCE: Finite alphabet Σ, finite set R of strings from Σ^*, and a positive integer K.

   QUESTION: Is there a string $w \in \Sigma^*$ such that each string $x \in R$ is a subsequence of w.

   (For example, $R = \{ab, cb, ca, ac\}$ and $K = 4$ is a positive instance by choosing $w = cabc$.)

9. **(All or nothing)** A language is defined to be in D^P if it is the intersection of a language in NP with one in co-NP. In other words, D^P is the set of languages which can be expressed as a set difference of two languages in NP.

   (a) Show UNIQUE-SAT is in D^P:

   INSTANCE: Boolean formula F

   QUESTION: Does F have exactly one satisfying assignment of its variables?

   (b) Find a D^P-complete language. Provide a proof.