CS 172: Computability and Complexity, Fall 2006 **S.A. Seshia**

Midterm 2

November 15. 2006

YOUR NAME:

Instructions:

This exam is *closed-book, open-notes.* Please turn off electronic devices: cell phones, laptops, PDAs, etc.

You have a total of 70 minutes. There are 4 questions worth a total of 100 points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it for a while and try another one.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page.

Do not turn this page until the instructor tells you to do so!

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

Problem 1: [True or False, with justification] (30 points)

For each of the following questions, state TRUE or FALSE. If TRUE, give a short proof. If FALSE, give a counterexample.

(a) The sentence

$$\forall x, y\ [(x \times y > 0) \rightarrow (x + y \geq 2)]$$

is in Th(**N** ,+, x) but not in Th(**N** ,+). (Recall that **N** denotes the set of natural numbers: {0,1,2,3,…}.)

(b) For every language L, it is true that $L \leq_m \overline{L}$.

(c) The following language L is decidable:

L = {<M> | M is TM and, for all inputs w, M halts on w within 42 steps}

Problem 2: (25 points)

A *jumpy Turing machine* (JTM) is just like the standard, single-tape, deterministic TM except for its transition function. On a transition, a JTM can move its head a finite, but arbitrary distance from its current location. Formally, the transition function is

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times (\{L, R\} \times \mathbf{N})$$

For example, $\delta(q_i, a) = (q_j, b, (L, n))$ will cause the JTM's head to move n places to the left (stopping at the left-most cell if the jump would cause the head to move off the tape).

Prove that every JTM has an equivalent standard, single-tape, deterministic TM. (Include all steps in your proof.)

Problem 3: (20 points)

Recall the *one way Turing machine* (OTM) from Homework 5. An OTM is a single-tape, deterministic Turing machine that has a "stay put" move *S* in place of the regular "move left" *L*. In other words, the transition function is of the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, S\}$$

Answer the *three* questions (a), (b), (c) below (remember to turn this page over!). You may use without proof results from previous homeworks and theorems covered in class and in Sipser's book.

(a) Let the language class $C = \{L \mid L$ is a language recognized by a OTM$\}$. Among the language classes you have studied in this class (regular, context-free, decidable, Turing-recognizable, etc.), which one is *C* equal to? Briefly justify your answer (no need to give a complete proof).

(b) Prove that the following language is undecidable:

$L_2 = \{<M> \mid M$ is a TM and $L(M) \in C\}$

(c) Prove that the following language is decidable:
$L_1 = \{<M> \mid M \text{ is an OTM and } L(M) = \emptyset\}$

Problem 4: (25 points)

Recall that a *clique* in a (undirected) graph G is a subset of vertices of G such that every two vertices in that subset are connected by an edge.

We discussed the following CLIQUE problem in class:

CLIQUE = {<G, k> | G is an undirected graph with a clique of size k}

Two variants of the CLIQUE problem are given below. For each language, state whether it is in P or is NP-hard. If P, give a polynomial-time algorithm to decide it. If NP-hard, give a reduction from CLIQUE or one of the NP-hard problems discussed in class.

(a) 11-CLIQUE = {<G> | G is an undirected graph with a clique of size 11}

(b) SpecialCLIQUE = {<G, v, k> | G is an undirected graph with a clique of size k that includes vertex v}