CS172, Luca Trevisan, Fall 2005, Midterm II, 75 minutes.

1. (35 Points) Prove that if M is Turing machine with q states, w is an input of length n, and M on input w moves right on all the first $n + q + 1$ steps, then M on input w does not halt.

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2. (35 Points) We use ℓ(x) to denote the length of a string x, and $K(x)$ to denote the smallest k such that there exists a pair ($\langle M\rangle$, w) such that $\ell(\langle M\rangle, w) \leq k$ and M on input w outputs x. We also define the language $R = \{ x : K(x) \geq \ell(x) \}$ of Kolmogorov random strings.

   Prove that if $L \subseteq R$ is recognizable language, then L must contain a finite number of strings. (Note that, in particular, this implies that R is not recognizable.)

   [Hint: prove that if $L \subseteq R$ is recognizable and infinite, then there is algorithm that on input an integer n outputs a string in R of length at least n, and then prove that this leads to a contradiction.]

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3. (30 Points) Prove that if A, B are two languages in NP, then $A \cup B$ and $A \cap B$ are also in NP.