# Midterm 2

5:00-7:00pm, 18 November

*Notes: There are three questions on this midterm. Answer each question in the space below it, using the back of the sheet to continue your answer if necessary. If you need more space, use the blank sheet at the end. Please write* clear, concise *answers. The questions vary quite a bit in difficulty, so if you are having problems with part of a question, leave it and try the next one. All three questions carry approximately equal credit; approximate point scores for each question part are shown.*

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**Your Name:**

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**1.** Which of the following statements are true? If the statement is true, give a *short* proof. If it is false, give a *simple* counterexample. You may assume without proof any result that was proved in class or on a Homework, provided you state it clearly.

(a) If $L$ is recognizable (r.e.) then its complement $\overline{L}$ is recognizable. *4pts*

(b) If $L_1$ and $L_2$ are both in NP, then $L_1L_2$ is in NP. [Recall that $L_1L_2$ denotes the language of all strings $xy$ where $x \in L_1$ and $y \in L_2$.] *4pts*

(c) If $L_1 \leq_m L_2$, then $L_2 \leq_m L_1$. *4pts*

(d) There exists a specific Turing machine $M$ for which the language $L = \{w : M \text{ accepts } w\}$ is undecidable. *4pts*

(e) The function $S(\langle M \rangle, n)$ is computable, where $S(\langle M \rangle, n)$ denotes the maximum space used by a Turing machine $M$ on any *halting* computation on inputs of length $n$. [The space used by a Turing machine is the maximum distance traveled by the head from the left-hand end of the tape. If $M$ fails to halt on all inputs of length $n$, then $S(\langle M \rangle, n)$ is defined to be zero.] *4pts*

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**2.** Consider the language

$$BOTH_{\mathrm{TM}} = \{ \langle M, w_1, w_2 \rangle \ : M \text{ accepts both } w_1 \text{ and } w_2 \}.$$

(a) Show that the language $BOTH_{\mathrm{TM}}$ is recognizable (r.e.). *4pts*

(b) By giving a reduction from the undecidable language $A_{\mathrm{TM}}$ to $BOTH_{\mathrm{TM}}$, prove that $BOTH_{\mathrm{TM}}$ is undecidable. *8pts*

(c) Is the language *8pts*

$$ONE\text{-}OF_{\mathrm{TM}} = \{ \langle M, w_1, w_2 \rangle \ : M \text{ accepts exactly one of } w_1 \text{ and } w_2 \}$$

recognizable? Justify your answer carefully.

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**3.** A boolean formula $\phi$ is in 2-*CNF* (2-conjunctive normal form) if it is the AND of a sequence of clauses, with each clause being the OR of exactly 2 literals. The formula is in *monotone* 2-CNF if there are no negated variables. The following formula is an example:

$$\phi = (x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_2 \vee x_4) \wedge (x_3 \vee x_4).$$

Note that every formula in monotone 2-CNF is trivially satisfiable by setting all the variables to TRUE (because there are no negations).

Consider the language

MON2-SAT = $\{ \langle \phi, k \rangle \; : \; \phi$ is in monotone 2-CNF and $\phi$ can be satisfied by setting only $k$ variables TRUE$\}$.

(a) Show that MON2-SAT is in NP. *4pts*

(b) Show that MON2-SAT is NP-complete. [Hint: Try a reduction from the Vertex Cover problem, which you may assume is NP-complete.] *10pts*

(c) Consider now the language *6pts*

MON2-SAT$_{10}$ = $\{ \langle \phi \rangle \; : \; \phi$ is in monotone 2-CNF and $\phi$ can be satisfied by setting only 10 variables TRUE$\}$.

Is MON2-SAT$_{10}$ likely to be NP-complete? Justify your answer carefully.

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**[The End!]**