Problem 1. (50 points) Determine which of the following two languages are context-free:

\[ A = \{ w \in \{0, 1\}^* : w = w^R \}, \]
\[ B = \{ w \in \{0, 1\}^* : w \neq w^R \}. \]

Here, \( w^R \) is the word \( w \) reversed. Prove your answers.

Problem 2. (100 points)

a. Consider a machine \( M \) and a state \( q \) of \( M \). The state \( q \) is dead if for all input words \( w \) and all runs \( r \) of \( M \) on input \( w \), the state \( q \) does not occur in \( r \). The state \( q \) is redundant if \( L(M) = L(M \setminus q) \), where \( M \setminus q \) is the machine that results from \( M \) by removing the state \( q \), as well as all transitions in and out of \( q \).

If \( q \) is dead, does it follow that \( q \) is redundant?
If \( q \) is redundant, does it follow that \( q \) is dead?

b. Given a machine \( M \) and a state \( q \) of \( M \), the dead-state problem asks if \( q \) is a dead state of \( M \).
Given a machine \( M \) and a state \( q \) of \( M \), the redundant-state problem asks if \( q \) is a redundant state of \( M \).

Consider the following six problems:

\[ D_{\text{NFA}}, D_{\text{PDA}}, D_{\text{TM}}: \] the dead-state problems for NFAs, PDAs, and TMs.
\[ R_{\text{NFA}}, R_{\text{PDA}}, R_{\text{TM}}: \] the redundant-state problems for NFAs, PDAs, and TMs.

For each of these six problems, determine which of the following four statements is true:

- **S1** The problem is recursive.
- **S2** The problem is r.e., but not recursive.
- **S3** The problem is co-r.e., but not recursive.
- **S4** The problem is neither r.e. nor co-r.e.

Prove your answers. You can use the membership, emptiness, universality, and equivalence problems for NFAs, PDAs, and TMs, and what we learned about them in class.