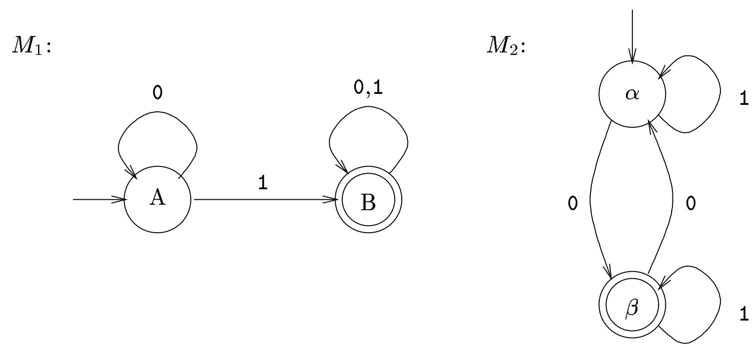
CS 172 — Fall 2000	Prelim 1
Computability and Complexity	**October 4, 2000**

Problem 1. (90 points) Let $M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$ be two finite automata over a common alphabet.

a. Construct a finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ such that $L(M)$ contains precisely the words in $L(M_1)$ which have a suffix in $L(M_2)$.

b. Apply your construction from part (a) to the following two automata.



c. Is the result from part (b) deterministic? If not, use the subset construction to determinize it.

d. Is the result from part (c) minimal? If not, use the minimization algorithm to minimize it.

e. Let A be the language of the automata from parts (b)–(c). What is the index of A? Describe each $\equiv_A$-equivalence class by a regular expression.

You can double check your result by thinking about which languages are accepted by the automata of part (b), and which language results from applying the operation of part (a) to these languages.

Problem 2. (60 points) For the two languages B_1 and B_2 defined below do the following.

If B_i is regular: Give a finite automaton that recognizes B_i.

If B_i is not regular: Give both a proof that B_i is not regular (use the pumping lemma or the fact that $\{0^n 1^n : n \geq 0\}$ is not regular) and a pushdown automaton that recognizes B_i.

a. B_1 is the set of all words in $\{0, 1\}^*$ that contain an equal number of occurrences of `00` and `11`. (Count overlapping occurrences; for example, `110001111` contains 2 occurrences of `00` and 4 occurrences of `11`.)

b. B_2 is the set of all words in $\{0, 1\}^*$ that contain an equal number of occurrences of `01` and `10`.

Before you begin to answer the questions, think about which words are in B_1 and B_2, and which are not, by listing a few examples.