# CS 170, Spring 2000
# Midterm #1
# Professor M. Clancy

This is an open-book test. You have approximately eighty minutes to complete it. You may consult any books, notes or other paper-based inanimate objects available to you. To avoid confusion, read the problems carefully. If you find it hard to understand a problem, ask us to explain it. If you have a question during the test, please come to the front or the side of the room to ask it.

This exam comprises 15% of the points on which your final grade will be based. Partial credit may be given for wrong answers. Your exam should contain six problems (numbered 0 through 5) on eight pages, with two more blank pages at the back of the exam. Please write your answers in the spaces provided in the test; in particular, we will not grade anything on the back of an exam page unless we are clearly told on the front of the page to look there.

Relax -- this exam is not worth having a heart failure about.

## Problem #0 - 1 point, 1 minute

Put your name on each page. Also make sure you have provided the information requested on the first page.

## Problem #1 - 4 points, 10 minutes

Suppose that the following statements initialize the union/find structure descibed in CLR section 22.3.

```
        for (k=0; k
```

```
On the next page show the data structures, including ranks of representative el
```

```
     Union (0,1);
     Union (2,3);
     Union (0,2);
     Union (4,5);
     Union (0,4);
```

```
Results of Union calls

            |
call        |  resulting data structure, including rank(s)
------------+-------------------------------------------------------------
Union(0,1); |
            |
```

```
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------------+---------------------------------------------------------------
Union(2,3); |
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------------+---------------------------------------------------------------
Union(0,2); |
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------------+---------------------------------------------------------------
Union(4,5); |
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            |
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------------+---------------------------------------------------------------
Union(0,4); |
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            |
```

## Problem #2 - 6 points, 15 minutes

This problem concerns an implementation of Prim's algorithm as given in CLR that maintains the priority queue as an

The three C declarations below are used to store the graph in adjacency list format. Some extra fields have been added

```
struct Graph {
   struct Vertex verticies[ ]; // The array of vertices
};

struct Vertex {
   int index;                  // The "name" of the vertex
   int key;       // Key value used to organize the priority queue
   struct QueueNode* qPtr;     // Pointer to a node in the queue
   struct NbrList* neighbors;  // Adjacency list.
};

struct NbrList {
   struct Vertex* nbr;         // The other endpoint of this edge
   int w;                      // The weight of this edge
   struct NbrList* next;
};
```

Elements of the priority queue, implemented as an unsorted singly linked list, are declared in C as follows.

```
struct QueueNode {
   struct Vertex* v;      // Pointer to the corresponding vertex
   struct QueueNode* next;// Ptr to the next priority queue element
};
```

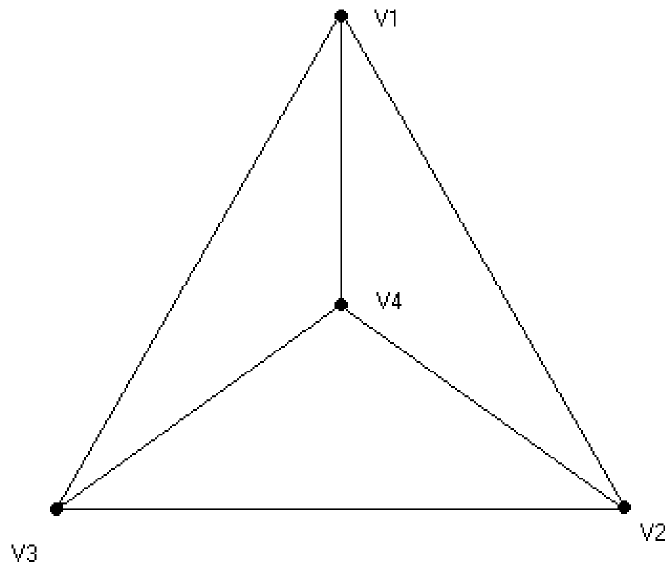
(The structs will all be classes in Java.)

### Part A

Give an estimate of the worst-case running time required to build the spanning tree for a graph of n vertices and e edge

### Part B

Assume that the contents of the queue are initially V1, V2, V3, V4. Supply edge weights for the graph below that prod




## Problem #3 - 3 points, 10 minutes

Give a tight estimate for the following recurrene, simplified as much as possible. Assume that values of T for small values of n are constant; that is, T(0) = ((1), T(1)=((1), T(2)=((1).

```
T(n) = 9T(n/3) + n^2 + n lg n
```

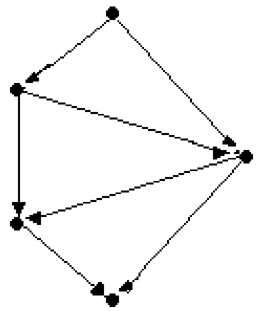
Explain your answer.

## Problem #4 - 8 points, 20 minutes

Prove that a graph G = (V,E) with no isloated vertices is strongly connected if and only if there is a circuit in G that inclued every edge at least once (and possibly more than once).

## Problem #5 - 8 points, 24 minutes

Give an efficient algorithm that, given a directed acyclic graph G = (V,E) and a vertex a in V, counts the number of paths from a to all other vertices. For example, there are five paths from a to b in the graph displayed below, namely aeb, aedb, aceb, acedb, and acdb.



Provide sufficient comments for us to understand how your algorithm works. An incorrect algorithm may earn you partial credit if we can understand it; if you know it won't work, provide a counterexample with your algorithm description.

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