CS170, Fall 2006, Midterm 1, Papadimitriou/Vazirani

NAME: _____

TA: _____

CS170 Midterm 1 October 4, 2006

- Be clear and concise.
- Total number of points is 50. So, you may use the number of points assigned to each problem as a rough estimate for the number of minutes you might allocate to the problem.
- Use the back pages as scratch paper.

1	
2	
3	
total	

1. **((20 points) Short questions)**

- How many numbers x are there between 1 and 90 such that x mod 5 = 1 and x mod 3 = 2?

- What is 3^{800} mod 15?

- In RSA, p = 7, q = 13, e = 5. Why is e = 5 an appropriate choice?

- What is the solution of $T(n) = T(n/2) + 1$, $T(1) = 0$?

- True or False? Explain very briefly: If ω is the n^{th} root of 1, then for any $k \leq n$

$$\sum_{i=0}^{n-1} \omega^{ik} = 0$$

- You wish to multiply the polynomials $x^2 + 2$ and $x^3 + x - 1$ using the FFT. At which points will you evaluate these polynomials? Write any complex numbers in your answer as a + ib.

2. **(10 points)** (a) How many lines will the following program print? Write the recurrence equation and give its solution in *O*(.) form.

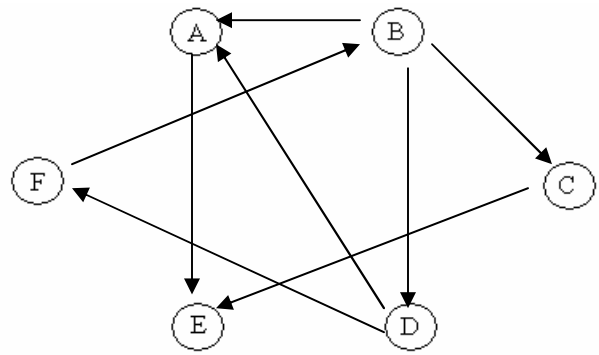
```
function mystery(n) (comment: n is a power of 2)

if n = 1: write("done!")
else:
mystery(n/2)
mystery(n/2)
mystery(n/2)
for i = 1 to n do:
for j = 1 to n do:
write("are we done yet?")
```

(b) How many, in θ(.) form, of these lines will read "done!"? Briefly justify.

3. **(20 points)**

- Perform depth-first search on this directed graph. Give the Previsit and Postvisit numbers of the nodes. Process nodes in alphabetical order (node A is visited first, and edge AB is visited before AD).



- A *vista vertex* of a directed graph is a vertex from which all other vertices are reachable. Does the graph above contain a vista vertex?

- True or False? (Explain briefly): If a graph has a vista vertex, it will be the last vertex from which `explore` is called in the main loop of depth-first search.

- Give a linear-time algorithm for telling if a graph has a vista vertex. Briefly justify its correctness. Why is it linear?