$\qquad$

| $(1)$ | 118 |
| :--- | :--- |
| $(2)$ | 120 |
| $(3)$ | 120 |
| $(4)$ | 116 |
| $(5)$ | 126 |
| TOTAL | 1100 |

## Final Examination

(O pen K atz, asynchronous \& test notes only, Calculators OK, 3 hours)
Include all final answers in locations indicated on these pages. Use space provided for all working. If necessary, attach additional sheets by staple at the end. BE SURE TO WRITE YOUR NAME ON EVERY SHEET.
(1) (18pts)
(a) F our chairs are placed in a row:


Each chair may be occupied (" 1 ") or empty (" 0 "). W rite a logic function $F(A, B, C, D$ ) which is " 1 " iff there are tw o (or more) adjacent chairs that are empty. Express F:
(i) A s a K arnaugh Map.
(ii) In minimum Sum-of-Products form using algebraic notation.
(iii) In Standard Sum-of-Products (minterm) form.
(iv) In Standard Product of Sums (maxterm) form.

1(a) 9pts
(i) $F(A, B, C, D)$ as a $K$ arnaugh $M$ ap:

(ii) In minimum SoP form, $F(A, B, C, D)=$ $\qquad$
(iii) In Standard SoP form, $F(A, B, C, D)=$ $\qquad$
(iv) In Standard PoS form, $F(A, B, C, D)=$ $\qquad$
(b) How many different combinational logic functions of two variables ( X and Y ) are there?

## 1(b) 2pts

Total number. of switching functions:
(c) G iven that:

$$
F(A, B, C, D)=\Sigma m(1,4,5,6,7,9,13,15)
$$

$U$ sing a K arnaugh Map:
(i) List all of the essential prime implicants.
(ii) How many prime implicants are there? L ist them.
(iii) How many implicants are there? J ust give the number, don't list them.
(iv) Implement the function using a single 8-to-1 M UX and two-input A ND gates only. A ssume variables and their complements are available. Be sure to label your M UX inputs.

1(c) 7pts

| $\lambda^{\mathrm{Al}}$ | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: |
| 00 |  |  |  |
| 01 |  |  |  |
| 11 |  |  |  |
| 10 |  |  |  |

(i) E ssential prime implicants: $\qquad$
(ii) Prime implicants: $\qquad$
(iii) T otal number of implicants: $\qquad$
(iv) Implementation:

Y our N ame: $\qquad$
(2) (20pts) Consider the clocked J-K flip-flip flop with asynchronous preset and clear as shown below:

(a) C omplete the follow ing timing diagram for Q output of the flip-flop. A ssume all internal delays are zero and that there are no setup or hold constraints.

2(a) (5pts)

(b) D erive the characteristic equation for the flip-flop, including the preset ( P ) and clear (C) inputs as well asJ and $K$. A ssume $P=C=0$ will never occur and treat $P$ and $C$ as synchronous inputs for this part of the problem.

2(b) (3pts)
$\mathbf{Q}_{\mathrm{n}+1}=$ $\qquad$
(c) Complete the timing diagram for the circuit shown below. N ote that the CK (clock) inputs on the two flip-flops are different. A ssume all internal delays are zero and that there are no setup and hold constraints.


Additional space for Problem 2

## Y our N ame:

$\qquad$
(3) (20pts)
(a) Construct a state graph for the circuit shown below ( X is the input and Z is the output.)


3(a) (6pts)

Additional space for Problem 3
(b) A sequential network has only one input ( $X$ ) and one output ( $Z$ ). D raw a M ealy state graph for the case $Z=1$ iff the total number of 1 's received is divisible by 3. (N ote: $0,3,6,9,12, \ldots$ are divisible by 3.)

3(b) (6pts)
(c) D raw the M ealy state graph for the network of (b) above if the total number of 1's received is divisible by 3 and the total number of 0 's received is an even number which is greater than zero. ( 9 states are sufficient.)
3(c) (8pts)

## Y our N ame:

(4) (16pts)
(a) D raw the schematic diagram for a minimal hazard-free realization (gates + gate inputs) of the following function using only 3-input N O R gates. A ssume variables and their complements are available.
$F(A, B, C, D)=\Sigma m(0,2,6,7,8,10,13)$
4(a) (6pts)

Additional spacefor Problem 4
(b) F or the flow table shown below, find a state assignment that avoids all critical races. A dditional states may be added as necessary, but use as few state variables as possible. A ssign the all-0's combination to state A. If any additional states are needed to prevent critical races, use your state assignment and adjacency map to explain why.


4(b) (10pts)
State assignment: $\qquad$
$\qquad$
$\qquad$
A djacency map:
$\qquad$
(5) (26pts)This problem concerns a base-(-2) (base-(minus-two)) adder. In CS150, we only considered number systems with positive bases (in particular, the binary, or base-2, system). For example, in a base3 system, the number $A B C D_{3}$ is equal to:
$A B C D_{3}=A * 3^{3}+B * 3^{2}+C * 3^{1}+D * 3^{0}$.
So for a base-(-2) system, EF G H -2 would be computed using:
EFGH ${ }_{-2}=E^{*}(-2)^{3}+F^{*}(-2)^{2}+G *(-2)^{1}+H^{*}(-2)^{0}$
(a) Convert the following decimal numbers to base-(-2). U se as many bits as you need.

5(a) (8pts)
$5_{10}=$
$-2 \quad 6_{10}=$
$-7_{10}=$
$11_{10}=$
(b) For the purpose of this problem, we define a number system as contiguous if, given any two integer values $m$ and $n$ which can be represented in the number system, there exists no integer value $x$, with $m<x<n$, that cannot be represented in the system. For example, a 1-bit decimal system is contiguous. It can represent the counting numbers $0-9$, and there are no numbers in between 0 and 9 which cannot be represented using one decimal digit. The binary number system (base 2 ) is al so contiguous.
(i) Is a four-bit base-(-2) system contiguous?
(ii) W hat are the maximum and minimum values (in decimal) that can be represented by a four-bit base-(-2) system?
(iii) Is an $n$-bit base-(-2) system contiguous, where n is any positive integer? J ustify your answer.

5(b) (6pts)
(i) $\qquad$
(ii) $\mathrm{M} \mathrm{aximum}_{10}=$ $\qquad$ Minimum $_{10}=$ $\qquad$
(iii) $\qquad$
(c) A dd the following numbers in base-(-2). Pay close attention to the carry-in and carry-out aspects because they are important in Part (d)

5(c) (2pts)
$001110_{-2}+001111_{-2}=$ $-2$
(d) D raw a truth table for a one-bit bit slice of a base-(-2) adder. A ssume one bit of carry-in. Y ou must determine the specification for the carry-out.

5(d) (3pts)
(e) R epeat part (d) above but with two carry-in bits.

## 5(e) (3pts)

(f) D raw the block-diagram for a 4-bit base-(-2) adder, using your one-bit bit slice from 5(e) above.

5(f) (4pts)

Additional Space for W orking Problems

