Problem #1
A Twisted Tail Ring counter is shown below. Show the state diagram, accounting for all possible states? Is this counter self-starting (self-correcting)? Explain.

Problem #2
A 4 --> 1 MUX (Multiplexer) shown below can be used to realize any 3-variable switching function with no added logic gates. In this problem we will try to find out whether a given 4-variable switching function $f(W, X, Y, Z)$ can be realized using a single 4 --> 1 MUX with no added gates.

Problem #2a
Given the function $f(W, X, Y, Z) = \sum m(2, 3, 4, 6, 7, 15) + \sum d(0, 5, 12, 13)$ and its K-map, is it possible to realize it using a single 4 --> 1 MUX by choosing $S_1 S_0 = W X$, $D_i$ (a member of) $\{0, 1, Y, \text{not } Y, Z, \text{not } Z\}$; $i = 0, 1, 2, 3$ (the complements of the input variables are available). If your answer is positive show the realization; if it's negative explain why.

Problem #2b
Repeat (a) for the choice $S_1 S_0 = Y Z$. 
Problem #2c
Repeat (a) for the choice \( S_1 S_0 = W Z \).

Problem #2d
How do you check with the aid of K-maps, the possibility of realizing a given 4-variable function using a single 4 --> 1 MUX?

Problem #2e
Estimate the percentage of 4-variable functions which can be realized using a single 4 --> 1 MUX.

Problem #3
State whether each of the following is true or false. If true prove or explain, if false give a counter example. A correct True or False answer with no explanation is worth only 1 point.

Problem #3a
No static hazards may occur when implementing a 4-variable logic function using a 4-to-16 decoder.

Problem #3b
The radix-4 modified Booth algorithm which examines three multiplier bits at once (with the rightmost bit serving as a reference bit) always results in the minimum number of add/subtract operations.

Problem #3c
A 2048 X 1 ROM can be used to implement an 8:1 MUX.

Problem #3d
Every finite state machine can be implemented as a Linear-Feedback-Shift-Register (LFSR).

Problem #3e
The following circuit can serve as a Flip-Flop in any sequential circuit.

![Flip-Flop Circuit](image)

**Problem #4**

Show an implementation of a circuit that multiplies the (unsigned) input number \( X = x_4 x_3 x_2 x_1 x_0 \) by 7 using only Full Adders (FAs) and inverters. In other words, the output number \( Z = z_{n-1} z_{n-2} ... z_1 z_0 \) satisfies \( Z = 7 \times X \). Determine the required number of output bits, \( n \), and show the implementation of your Multiply-by-7 circuit using as few FAs and inverters as possible.

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*Posted by HKN (Electrical Engineering and Computer Science Honor Society)*
*University of California at Berkeley*

*If you have any questions about these online exams please contact examfile@hkn.eecs.berkeley.edu.*