# University of California at Berkeley <br> College of Engineering <br> Department of Electrical Engineering and Computer Science 

EECS 150
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Fall 2000

## FIRST MIDTERM EXAMINATION

Tuesday, 3 October 2000

INSTRUCTIONS—READ THEM NOW! All work is to be done on these pages. Partial credit is given only if we can evaluate your approach: indicate your assumptions and write as neatly as possible. Points are assigned to problems based on our estimates of how long they should take- 1 point equals 1 minute. PACE YOURSELF ACCORDINGLY: it is better to get partial credit on all of the problems than to complete a handful of them. This is a closed book examination. You will not need a calculator or other information appliance. You may use a single $8.5^{\prime \prime}$ by 11 " piece of paper with prepared notes. Write your name and Student ID Number at the top of each examination page.

Please refrain from discussing the examination after you have taken it. A small number of students are taking a late examination due to special circumstances.

It is a sad fact of life that cheating sometimes happens. It will not be tolerated. By signing below, you assert that all of the work included herein is your own, and that you understand the harsh penalties that will be imposed should cheating be detected-a 0 on the examination, and a letter of reprimand to your file:
$\qquad$
(Name—Please Print!)

| QUESTION | POINTS ASSIGNED | POINTS OBTAINED |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 15 |  |
| 3 | 20 |  |
| 4 | 15 |  |
| 5 | 15 |  |
| 6 | 10 |  |
| 7 | 90 |  |
| TOTAL |  |  |

$\qquad$ SID: $\qquad$

## Question 1. Terminology

(i) Prime Implicants (5 Points): In a 3-variable K-map, show a function and its prime implicants that illustrates the fact that there is more than one equally good minimized Sum of Products realizations of the function.
(a) Fill in the K-map for $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})$ below.
(b) Identify the prime implicants by suitable circlings.
(c) Select two different covers from the prime implicants, each with the same number of product terms and literals.


First minimized implementation of $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=\square$
Second minimized implementation of $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=\square$
(ii) Product of Sums versus Sums of Products (5 Points): Give a K-map and an example function whose minimized PoS form has fewer literals in it than its minimized SoP form.
(a) Fill in the K-map for $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})$ below.
(b) Indicate the minimized SoP and PoS realizations you obtain.
(c) How many literals is in each realization?


Minimized SoP realization of $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=$ $\square$
Number of literals is: $\qquad$

Minimized PoS realization of $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C})=$ $\square$

Number of literals is: $\qquad$

Student Name: $\qquad$ SID: $\qquad$

Question 2. Minimization (15 Points)
Given the following four four-variable functions:

$$
\begin{aligned}
\mathrm{W}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}) & =\sum \mathrm{m}(0,2,4,5,8,10,12,13) \\
\mathrm{X}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}) & =\sum \mathrm{m}(0,2,5,7,13,15) \\
\mathrm{Y}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}) & =\sum \mathrm{m}(4,5,6,8,10,12,13,14) \\
\mathrm{Z}(\mathrm{~A}, \mathrm{~B}, \mathrm{C}, \mathrm{D}) & =\sum \mathrm{m}(5,6,7,8,10,13,14,15)
\end{aligned}
$$

Use K-maps to independently minimize the functions in Sum of Products form.

$\square$
$\mathrm{X}=\square$
$\square$
$\mathrm{Z}=\square$

The number of unique product terms is: $\qquad$

Now resolve for $\mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}$ when the target of implementation is a Programmable Logic Array (PLA):

$\square$
$\square$

The number of unique product terms is: $\qquad$
$\qquad$ SID: $\qquad$

Question 3. Minimization (20 Points)
Consider the following function: $\mathrm{F}(\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D})=\Sigma \mathrm{m}(0,5,6,9,12,15)$
(a) What is the minimized Sum of Products form? How many literals does it have?


Literal Count is $\qquad$
(b) What is the minimized Product of Sums form? How many literals does it have?


Literal Count is $\qquad$
(c) In addition to AND, OR, Complement (NOT) functions, assume that you can also use the XOR function as a primitive gate. They cost no more to use than "simple gates" like AND and OR. How do you obtain an implementation with a minimized literal count now? Write down your answer as a twolevel Boolean expression using XOR, AND, OR, NOT functions.
$\square$

Literal Count is $\qquad$
Show your work below:
$\qquad$ SID: $\qquad$

Question 4. Circuits with Feedback (15 points)
Consider the following circuit schematic and timing waveform. Assume that all gates have identical gate delays. Each time division in the waveform diagram represents a gate delay. At time T0, the switch has been open long enough for the signals to have reached a steady initial state. At T1, the switch closes. Fill in the waveform diagram.



What kind of periodic signal does this circuit generate? $\qquad$
$\qquad$

Question 5. Multiplexer Design (5 Points)
Design a subsystem using multiplexer components that can do the following function:
Data Inputs: A, B, C, D
Control Inputs: S1, S0
Outputs: W, X, Y, Z
The behavior of the system is described by the following "functional" truth table:
Rotate Inputs Right
Rotate Inputs Left
Arithmetic Shift Inputs Right
Arithmetic Shift Inputs Left

|  |  | W X Y Z |
| :---: | :---: | :---: |
| 0 | S0 | W A B C |
| 0 | 0 | 1 |
| 1 | B C D A |  |
| 1 | 0 | 0 A B C |
| 1 | 1 | B C D 0 |

Draw a wiring diagram for your implementation below:

$\qquad$

Question 6. Multiplexer Implementation (15 Points)
Given the five variable function $F(A, B, C, D, E)=\Sigma m(0,1,3,4,8,9,10,11,12,14,16,20,21,23,26,30)$, show how to implement this using a single $8: 1$ multiplexer and no other logic gates (you may assume that variables and their complements are available at no cost to your implementation). HINT: Use A, B, E as the multiplexer control inputs. Show the wiring below:


Show your approach below:
$\qquad$

Question 7. Design Problem (10 Points)
It is well known from the X-Files that Aliens have two arms and three fingers on each hand. So it is not so surprising that they have a base 6 number system.

Design a digital subsystem that takes as input a "binary coded hexary" (BCH) digit (i.e., 0 through 5 is represented by the binary numbers 000 through 101) and a mode input that outputs the BCH digit plus 1 when mode $=0$ and $\mathrm{BCH}-1$ when mode $=1$.
(a) Identify your inputs and outputs. Draw a block diagram:

## BCH

Increment/
Decrement
By 1
(b) State your assumptions about the behavior of the circuit. Document your understanding of the function with a truth table:
(c) Implement it in minimized Sum of Products form. Draw filled in K-maps, circled implicants, and the minimized Boolean equations for your outputs:

