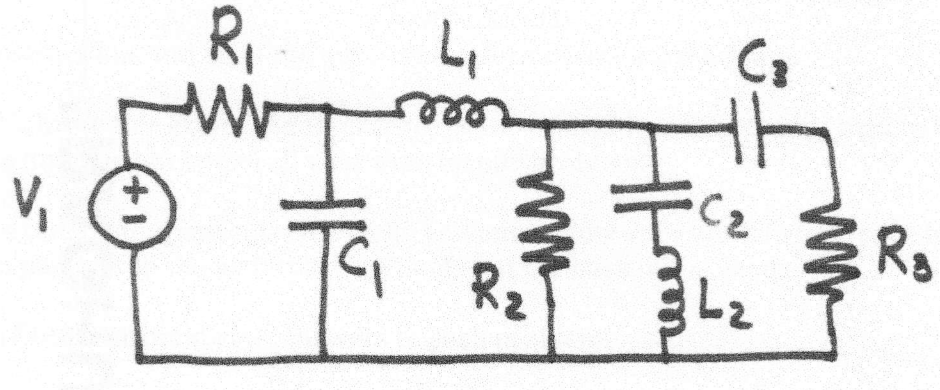
HAL: I'm sorry, Dave. I'm afraid I can't do that.
Dave Bowman: What's the problem?
HAL: I think you know what the problem is just as well as I do.
- *2001: A Space Odyssey*, Arthur C. Clarke

Problem 1 *(15 points)*

For all of the problems below,

$$V_1 = \begin{matrix} 0\,V \; for \; t<0 \\ 1\,V \; for \; t \geq 0 \end{matrix}$$

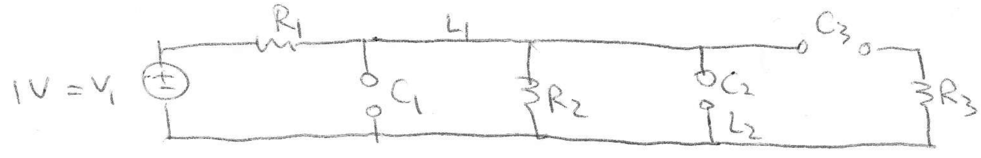
a) Consider the circuit below. *(5 points)*



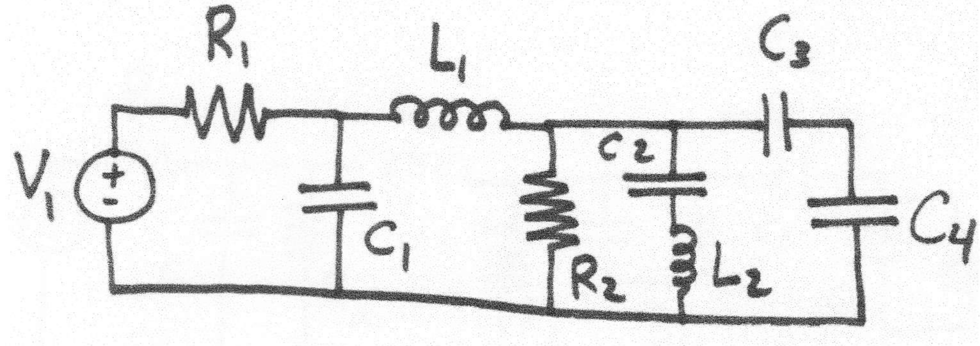
Provide a symbolic expression for the following at t = ∞:

Solution:	
$v_{C1} = \frac{R_2}{R_1+R_2}$ [V]	$i_{C1} = 0$
$v_{L1} = 0$	$i_{L1} = \frac{1}{R_1+R_2}$ [A]
$v_{C2} = \frac{R_2}{R_1+R_2}$ [V]	$i_{C2} = 0$
$v_{L2} = 0$	$i_{L2} = 0$
$v_{C3} = \frac{R_2}{R_1+R_2}$ [V]	$i_{C3} = 0$

at t=∞, C open, L short



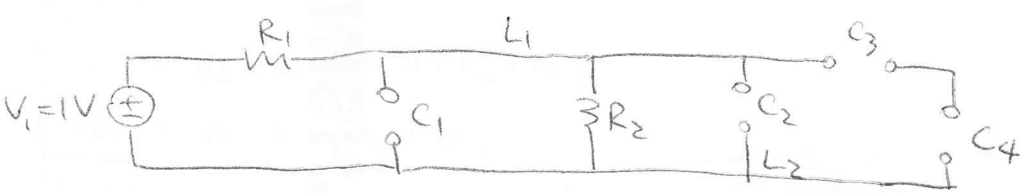
b) Consider the circuit below. *(5 points)*



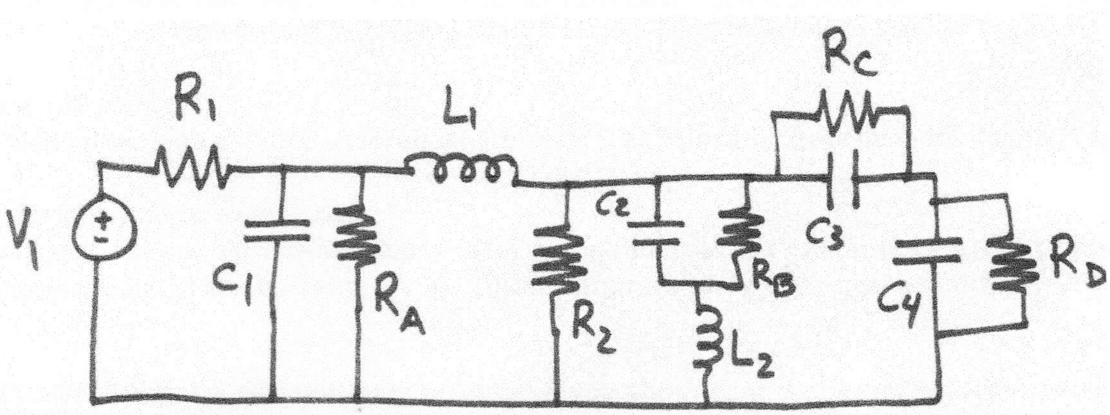
Provide a symbolic expression for the following at t = ∞:

Solution:	
$v_{C1} = \frac{R_2}{R_1+R_2}$ [V]	$i_{C1} = 0$
$v_{L1} = 0$	$i_{L1} = \frac{1}{R_1+R_2}$ [A]
$v_{C2} = \frac{R_2}{R_1+R_2}$ [V]	$i_{C2} = 0$
$v_{L2} = 0$	$i_{L2} = 0$
$v_{C3} = 0\text{ V}$ OR $\frac{R_2}{R_1+R_2} \cdot \frac{C_4}{C_3+C_4}$ [V]	$i_{C3} = 0$

$t = \infty$



c) Since real capacitors have a leakage resistance, let's add that in. Consider the circuit below. *(5 points)*



Provide a symbolic expression for the following at t = ∞:

Solution:

$V_{C1} = \frac{R_{th1}}{R_{th1}+R_1}$ [V]	$i_{C1} = 0$
$V_{L1} = 0$	$i_{L1} = \frac{1}{R_1+R_{th1}} \cdot \frac{R_A}{R_A+R_{th2}}$ [A]
$V_{C2} = \frac{R_{th1}}{R_{th1}+R_1}$ [V]	$i_{C2} = 0$
$V_{L2} = 0$	$i_{L2} = \frac{R_{th1}}{(R_{th1}+R_1)\,R_B}$ [A]
$V_{C3} = \frac{R_{th1}}{R_{th1}+R_1} \cdot \frac{R_C}{R_C+R_D}$ [V]	$i_{C3} = 0$

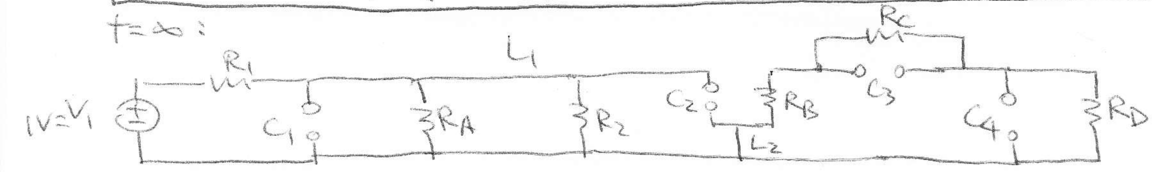
Extra Space

where

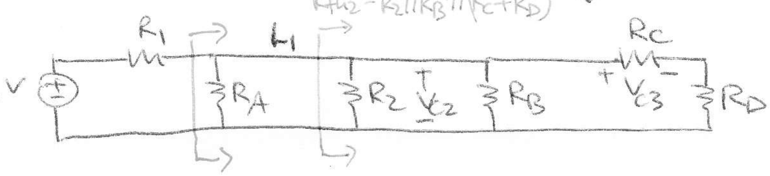
$R_{th1} = R_A \parallel R_2 \parallel R_B \parallel (R_C+R_D)$

$R_{th2} = R_2 \parallel R_B \parallel (R_C+R_D)$

$t=\infty$:



$R_{th2} = R_2 \parallel R_B \parallel (R_C+R_D)$ ⇓



$R_{th1} = R_A \parallel R_2 \parallel R_B \parallel (R_C+R_D)$

There is a theory which states that if ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable. There is another theory which states that this has already happened. — Douglas Adams, *The Restaurant at the End of the Universe*

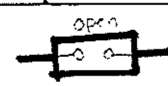
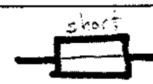
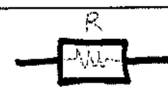
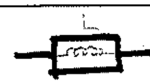
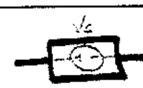
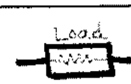
Problem 2 *(30 points)*

In each section below, you will find a circuit where several components have been left "blank" for you to fill in.

- The op amp is ideal in each case.
- In all cases, the opamp is connected to ideal voltage sources of value V_1 and V_2, where shown.
- Voltages are in [V] and currents are in [A]; constants are in the appropriate units.
- There should be no need for positive feedback in your solutions.

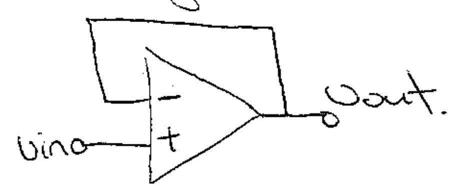
IMPORTANT: Partial credit will only be given if you show your work and solution (we can't give you partial points for blank circuits!).

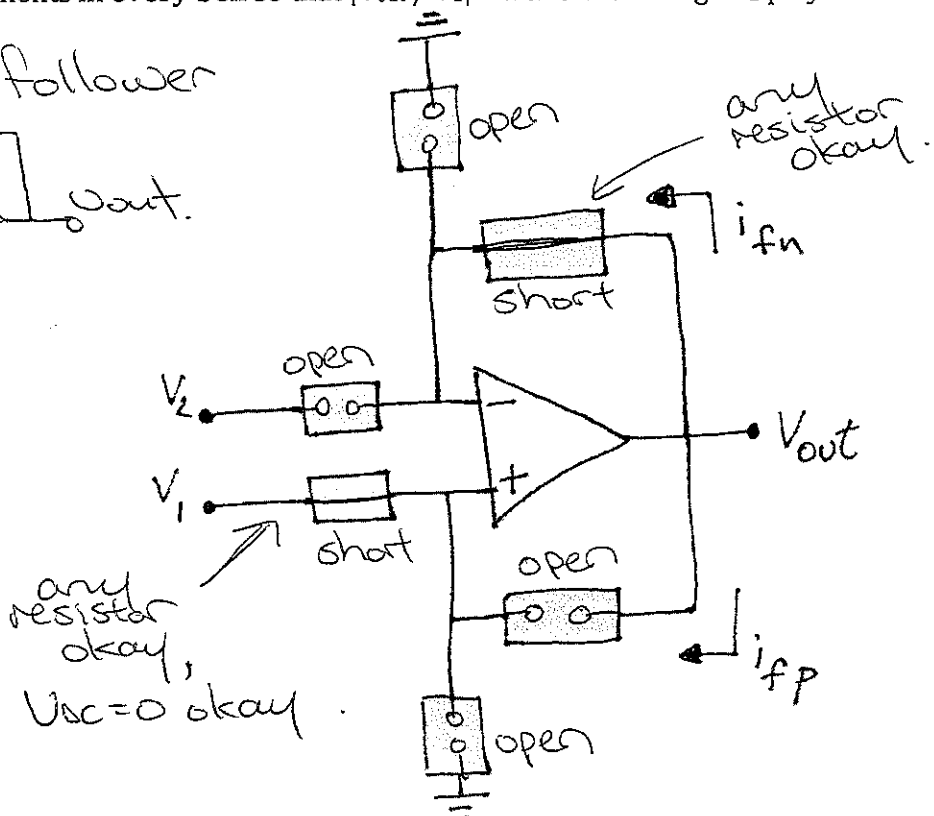
You have at your disposal as many of the following components as you like:

an open circuit	
a short circuit	
a resistor (you choose the value)	
a capacitor	
an inductor	
a DC voltage source (you choose *a fixed value*)	
a load (whose value is unknown but high enough that the amp can drive it)	

a) Add components in **every box** so that $|V_{out} / V_1| = 1$ and the voltage V_2 plays no role. *(7.5 points)*

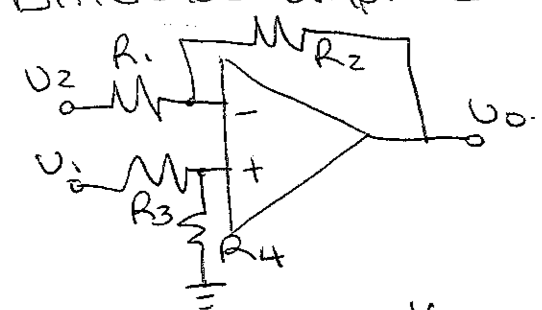
Voltage follower

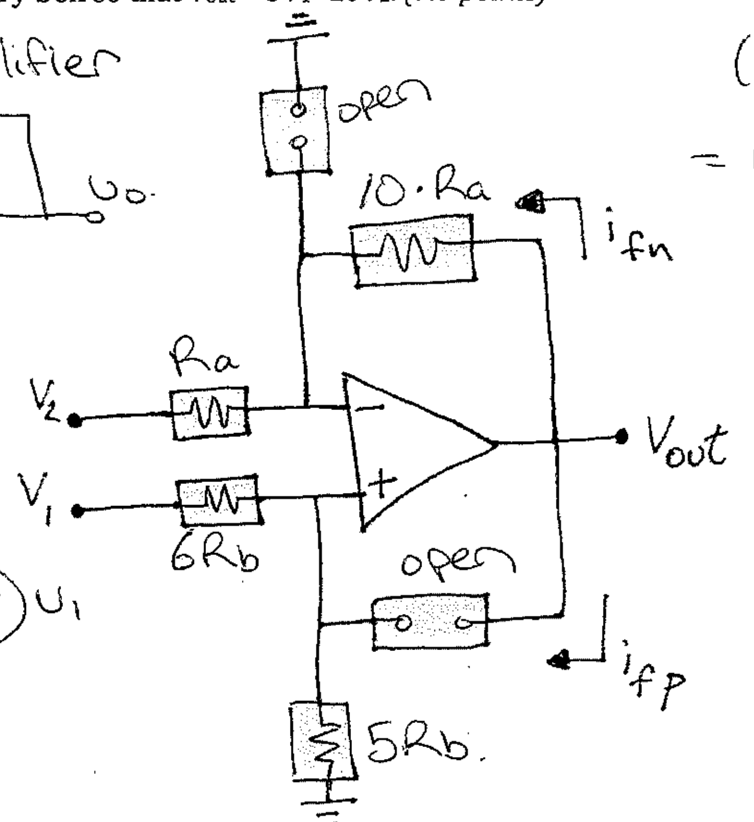




b) Add components **every box** so that $V_{out} = 5V_1 - 10V_2$. *(7.5 points)*

Difference amplifier





$$\left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)$$

$$= (11)\frac{1}{1+\frac{R_3}{R_4}} = 5$$

$$\left(1+\frac{R_3}{R_4}\right) = \frac{11}{5}$$

$$\boxed{\frac{R_3}{R_4} = \frac{6}{5}}$$

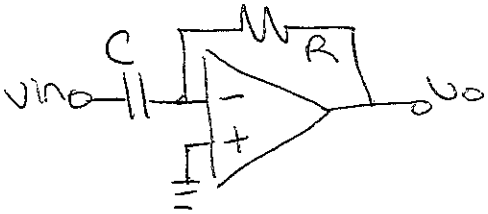
$$v_o = \left(1+\frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_1 - \left(\frac{R_2}{R_1}\right)V_2$$

$$= 5V_1 - 10V_2.$$

$$\boxed{\frac{R_2}{R_1} = 10}$$

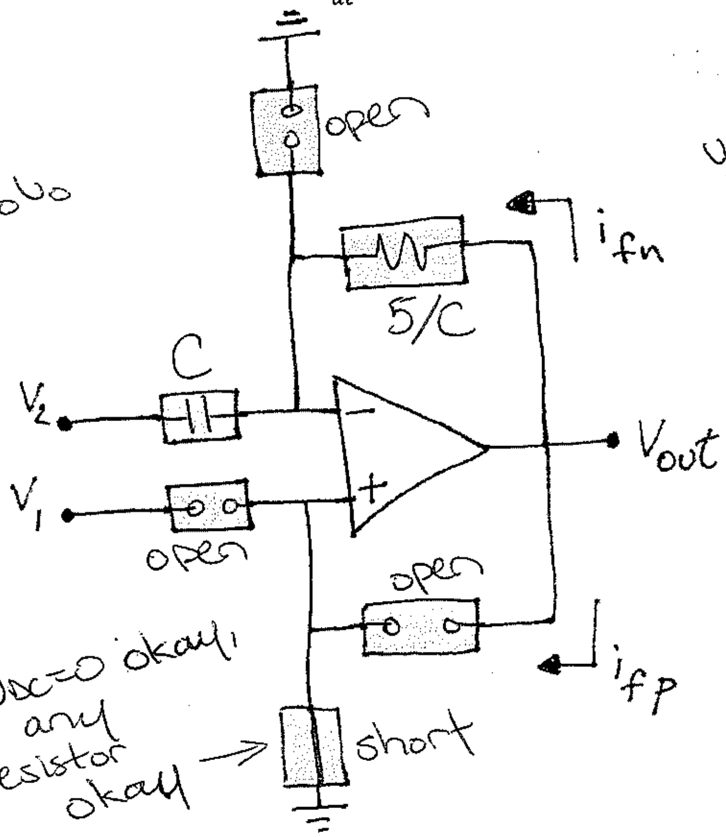
c) Add components **every box** so that $V_{out} = -5\frac{dV_2}{dt}$ and the voltage V_1 plays no role. *(7.5 points)*

Differentiator

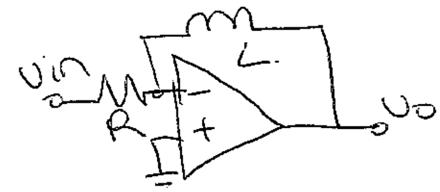


$v_o = -RC\frac{dv_{in}}{dt}$

$\boxed{RC=5}$



OR



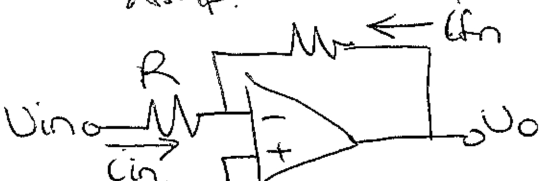
$v_o = -\frac{L}{R}\frac{dv_{in}}{dt}$.

$\boxed{\frac{L}{R}=5}$

d) Add components **every box** so that ~~$i_{fn} = 10V_2$~~ and the voltage V_1 plays no role. *(7.5 points)*

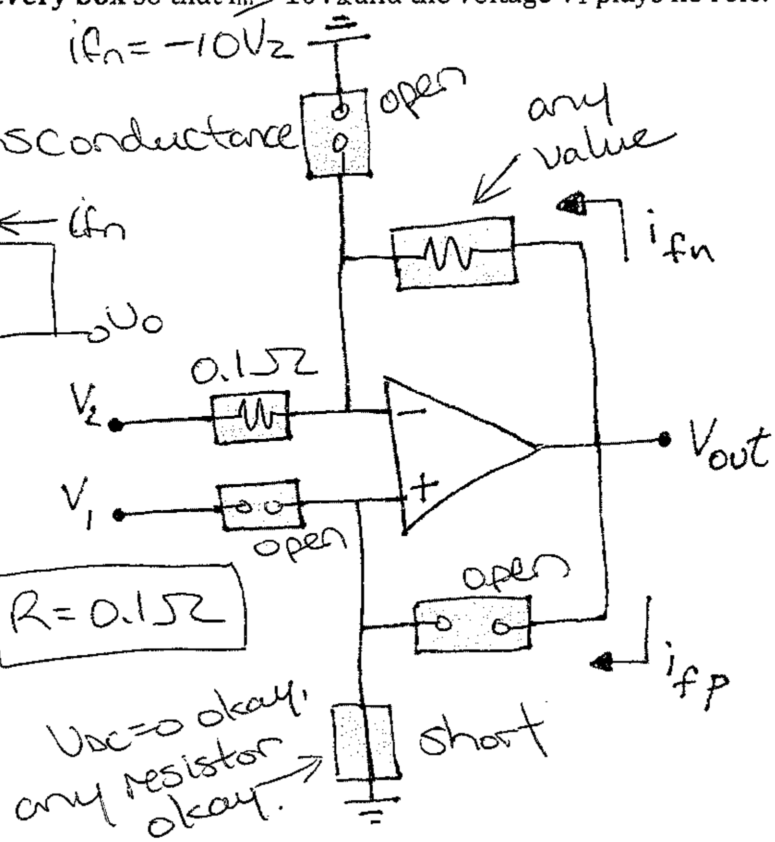
$i_{fn} = -10V_2$

Inverting transconductance Amp.



$i_{fn} = -i_{in} = \frac{-v_{in}}{R}$

$\frac{1}{R} = 10 \Rightarrow \boxed{R = 0.1\Omega}$

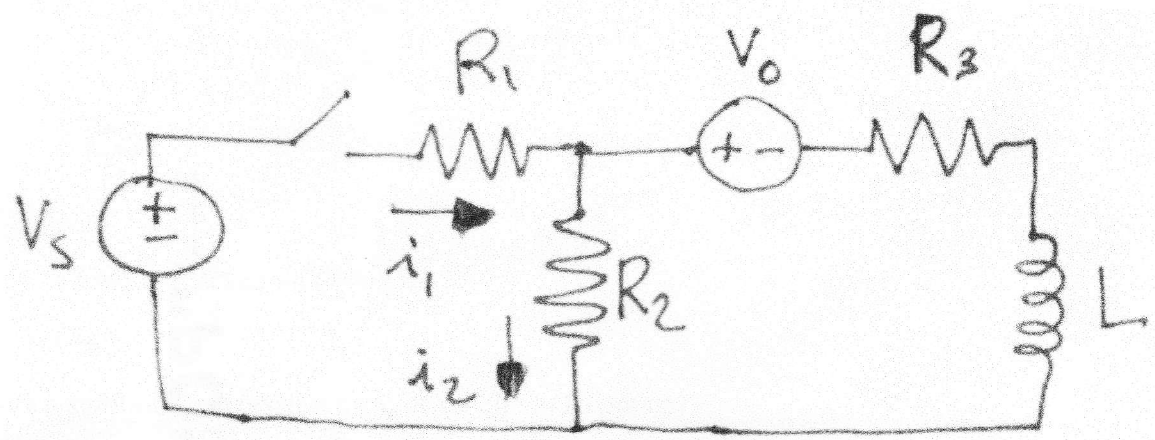


"Minor magicians take pains to fit this traditional wizardly bill. By contrast, the really powerful magicians take pleasure in looking like accountants."
— Jonathan Stroud, *The Amulet of Samarkand*

Problem 3 *(25 points)*

Consider the circuit below.

The switch is closed until t = 20 s, then opened. $V_s, V_o, R_1, R_2, R_3, L$ are given.

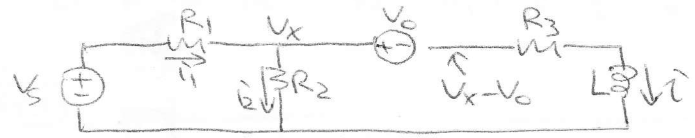


a) Provide a symbolic expression for i_1 at t < 20 s? *(2.5 points)*

Solution:

$$i_1 = \frac{V_s}{R_1} - \frac{1}{R_1}\left(\frac{V_s}{R_1} + \frac{V_o}{R_3}\right)\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}, \quad t<20\,s$$

t<20



L is a short.

$$\frac{V_s - V_x}{R_1} = \frac{V_x}{R_2} + \frac{V_x - V_o}{R_3}$$

$$\frac{V_s}{R_1} + \frac{V_o}{R_3} = V_x\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

$$\therefore V_x = \left(\frac{V_s}{R_1} + \frac{V_o}{R_3}\right)\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$\rightarrow i_1 = \frac{V_s - V_x}{R_1}$$

$$= \frac{V_s}{R_1} - \frac{1}{R_1}\left(\frac{V_s}{R_1} + \frac{V_o}{R_3}\right)\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

b) Provide a symbolic expression for i_2 at t < 20 s? *(2.5 points)*

Solution:

$$i_2 = \frac{1}{R_2}\left(\frac{V_s}{R_1} + \frac{V_o}{R_3}\right)\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}, \quad t<20\,s$$

$$i_2 = \frac{V_x}{R_2}$$

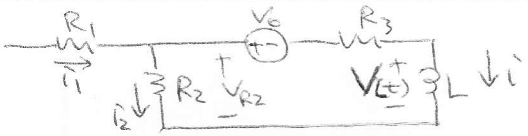
$$i(20) = i = i_1 - i_2 = \frac{V_s}{R_1} + \left(-\frac{1}{R_2} - \frac{1}{R_1}\right)\left(\frac{V_s}{R_1} + \frac{V_o}{R_3}\right)\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

c) Provide a symbolic expression for i_1 at $t \geq 20$ s? *(10 points)*

Solution:

$i_1 = 0\,A$

$t \geq 20s,$



$i(\infty) = -\frac{V_o}{R_2+R_3}$

$\tau = \frac{L}{R_2+R_3}$

$i(t) = i(\infty) + [i(20) - i(\infty)]e^{-(t-20)/\tau}$

$i_2 = -i(t)$

d) Provide a symbolic expression for i_2 at $t \geq 20$ s? *(10 points)*

Solution:

$i_2 = -i(\infty) + [i(\infty) - i(20)]\, e^{-(t-20)/\tau}$, $t \geq 20s$

$i(\infty) = -\frac{V_o}{R_2+R_3}$

$i(20) = \frac{V_s}{R_1} - \left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{V_s}{R_1}+\frac{V_o}{R_3}\right)\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$

$\tau = \frac{L}{R_2+R_3}$

"And yet, will we ever come to an end of discussion and talk if we think we must always reply to replies? For replies come from those who either cannot understand what is said to them, or are so stubborn and contentious that they refuse to give in even if they do understand."
— St. Augustine of Hippo, *City of God*

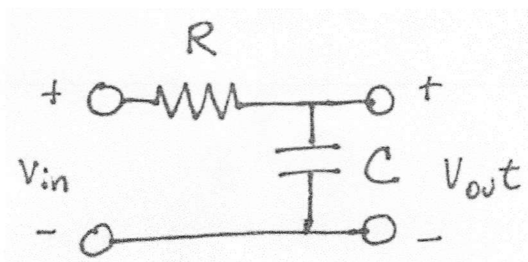
Problem 4 *(30 points)*

For all the circuits below,

$$V_{1\,\text{in}} = \begin{cases} 0\,V \ for\ t<0 \\ 1\,V \ for\ t \geq 0 \end{cases}$$

Provide an expression for $v_{out}(t)$ for $t \geq 0$.

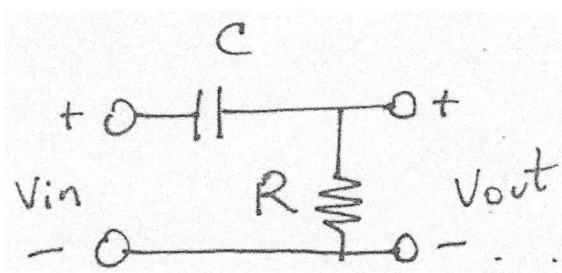
a) *(5 points)*



Vout(t)= $1-e^{-t/(RC)}$ [V]

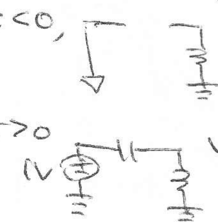
$V_{out}(0)=0V$
$V_{out}(\infty)=1V$

b) *(5 points)*



$t<0,$ $V_c(0)=0$

$t>0$ $V_c(\infty)=1V$



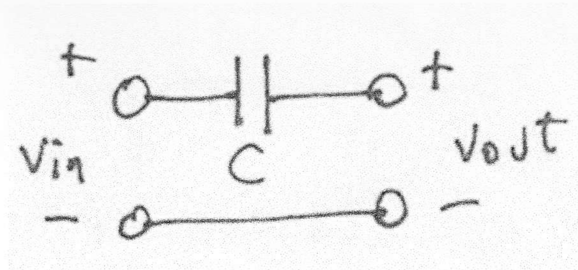
$V_c(t)=1-e^{-t/\tau}$

$V_{out}=V_{in}-V_c$

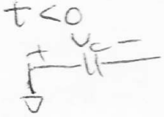
$=1-1+e^{-t/\tau}$

Vout(t)= $e^{-t/\tau} = e^{-t/(RC)}$ [V]

c) *(5 points)*

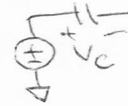


$t<0$



$V_c(0)=0$

$t>0$



$V_c(\infty)=0$

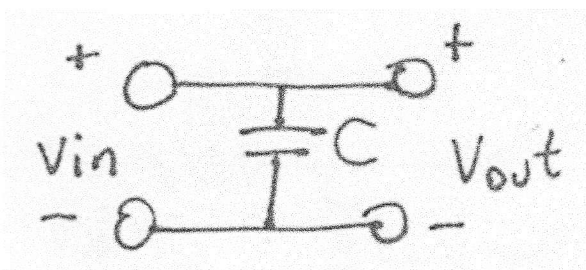
$\therefore V_c(t)=0$

$V_{out} = V_{in} - V_c$

$= 1V$

Vout(t)= 1V

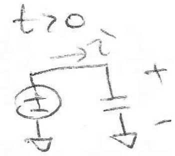
d) *(5 points)*



$t<0$

$V_c(0)=0.$

$t>0$



$V_c(\infty)=1V$

$R=0$

$\tau=RC=0.$

$V_{out}=V_c$

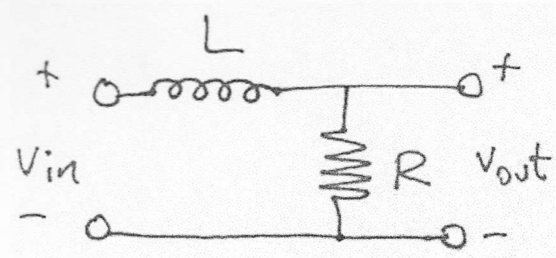
$= 1 + [0-1]e^{-t/0}$

$= 1V$

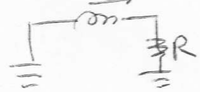
Vout(t)= 1V.

Note:
- This is only true when $i = \delta(t)\cdot C$. [Dirac delta function times C]
- $\delta(t)$ is an infinitely high, infinitely thin spike at $t=0$; $\int_{-\infty}^{\infty}\delta(t)\,dt \stackrel{\text{def}}{=} 1.$
- $V = \frac{1}{C}\int_{-\infty}^{t} i\,dt = \frac{1}{C}\int_{-\infty}^{t}\delta(t)\cdot C\,dt = 1\,V.$
- No such $\delta(t)$ in real life.

e) (5 points)

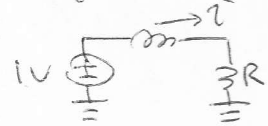


$t<0$



$i(0)=0$

$t>0$



$i(\infty)=\frac{1}{R}$ A

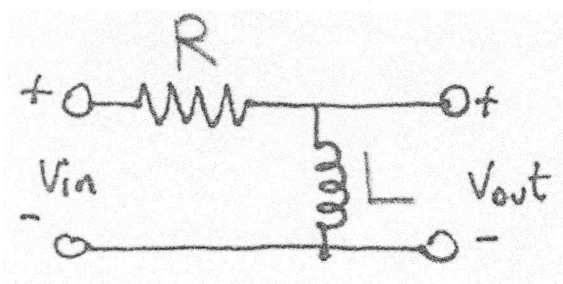
$i(t)=\frac{1}{R}-\frac{1}{R}e^{-t/\tau}, \tau=\frac{L}{R}$

$V_{out}(t)=i(t)\cdot R$

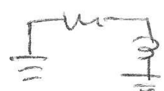
$=1-e^{-t/\tau}$

V$_{out}$(t)= $1-e^{-tR/L}$ [V]

f) *(5 points)*

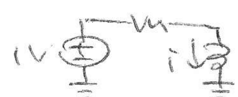


$t<0$



$i(0)=0$

$t>0$



$i(\infty)=\frac{1}{R}$ [A]

$i(t)=\frac{1}{R}-\frac{1}{R}e^{-t/\tau}, t>0$

$\tau=\frac{L}{R}$

$V_{out}(t)=L\frac{di}{dt}$

$=L\left(\frac{1}{R}\frac{1}{\tau}e^{-t/\tau}\right)$

$=e^{-t/\tau}$

V$_{out}$(t)= $e^{-tR/L}$ [V]