

**EECS 40, Spring 2007**  
**Prof. Chang-Hasnain**  
**Midterm #1**

February 21, 2007  
 Total Time Allotted: 80 minutes  
 Total Points: 100

1. This is a closed book exam. However, you are allowed to bring one page (8.5" x 11"), single-sided notes.
2. No electronic devices, i.e. calculators, cell phones, computers, etc.
3. SHOW all the steps on the exam. Answers without steps will be given only a small percentage of credits. Partial credits will be given if you have proper steps but no final answers.
4. Draw BOXES around your final answers.
5. **Remember to put down units.** Points will be taken off for answers without units.

Last (Family) Name: Perfect

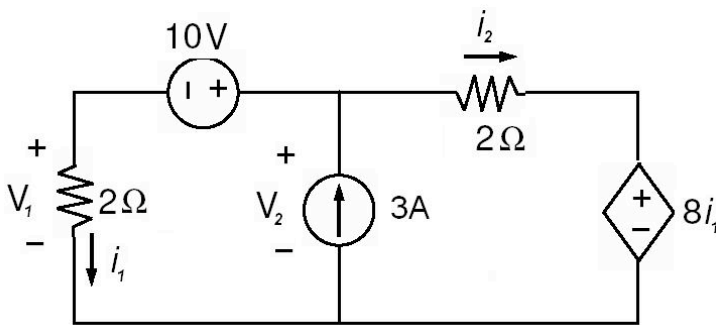
First Name: Peter

Student ID: 00000000 Discussion Session: \_\_\_\_\_

Signature: \_\_\_\_\_

|                     |  |
|---------------------|--|
| <b>Score:</b>       |  |
| Problem 1 (26 pts)  |  |
| Problem 2 (28 pts): |  |
| Problem 3 (32 pts)  |  |
| Problem 4 (14 pts)  |  |
| Total               |  |

## 1. (26 pts) Circuit Analysis



(a) (4 pts) Express  $i_1$  in terms of  $V_1$  and constants given in this problem.

$$V_1 = 2 i_1$$

(b) (4 pts) Express  $i_2$  in terms of  $V_1$  AND  $V_2$  and/or constants given in the problem.

$$i_2 = 3 - V_1/2$$

(c) (10 pts) Write two equations in  $V_1$  and  $V_2$  that can be used to solve the circuit (Hint: Use KCL or KVL.).

$$10 + V_1 - V_2 = 0$$

$$2 \times (3 - V_1/2) + 8 \times (V_1/2) - V_2 = 0$$

(d) (8 pts) Solve for  $V_1$ ,  $V_2$ ,  $i_1$  and  $i_2$ .

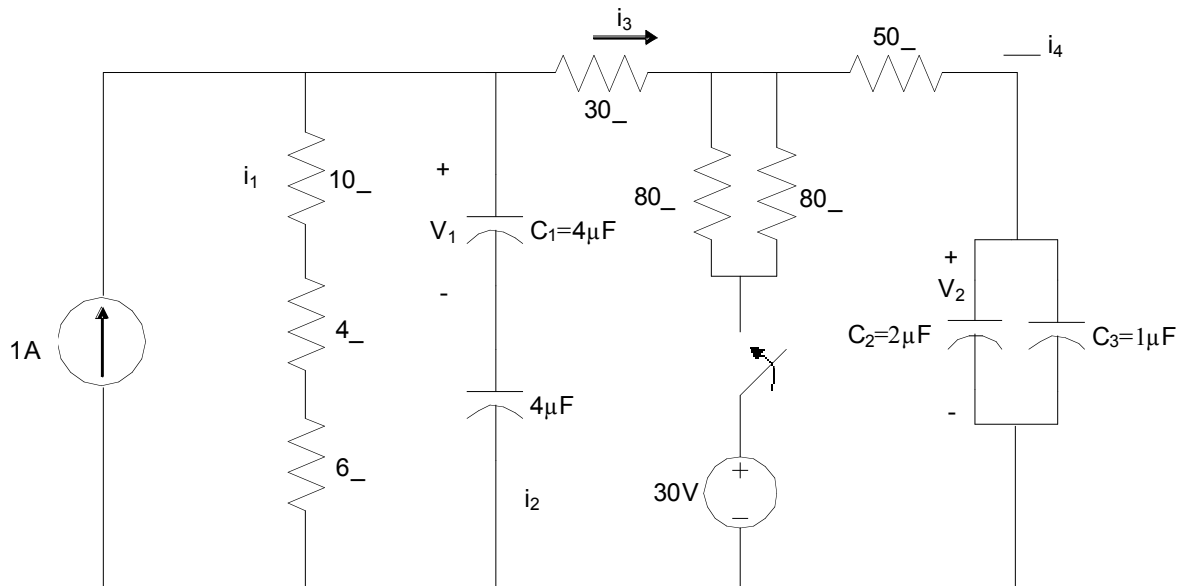
$$V_1 = 2 \text{ V}$$

$$V_2 = 12 \text{ V}$$

$$i_1 = 1 \text{ A}$$

$$i_2 = 2 \text{ A}$$

2. (27 pts) First-Order Circuit. **Remember to put down units.**



a) (12 pts) At  $t=0$ , the switch closes. Find the indicated current and voltages at  $t=0^-$  s, immediately **BEFORE** the switch closes. Note the current source has been active for a long time before the switch closes.

Provide the steps or explanation for your answers, e.g. using KCL/KVL, etc.

|       |     |
|-------|-----|
| $i_1$ | 1A  |
| $i_2$ | 0A  |
| $i_3$ | 0A  |
| $i_4$ | 0A  |
| $V_1$ | 10V |
| $V_2$ | 20V |

All capacitors are open circuits, all currents but  $i_1$  are 0.  $i_1$  is the same as the current source.

$$V_1 = \left( \frac{(10\Omega + 4\Omega + 6\Omega) * 1A}{2} \right) = 10Volts$$

$$V_2 = (10\Omega + 4\Omega + 6\Omega) * 1A = 20Volts$$

b) (3 pts) At  $t=0^+$  s, immediately **AFTER** the switch closes. Which quantities will be different?

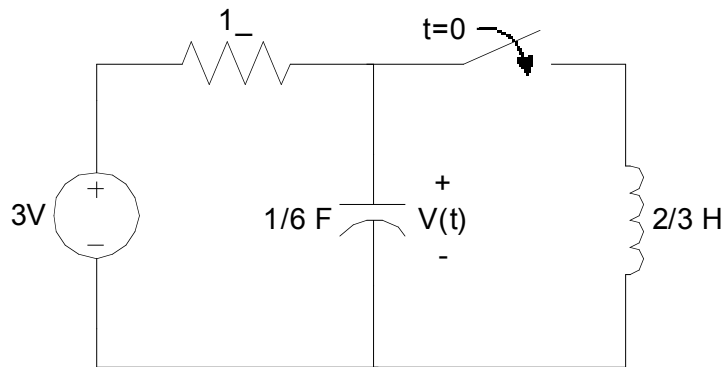
All the currents may be different, so  $i_1$ ,  $i_2$ ,  $i_3$ , and  $i_4$  may change (in fact  $i_1$  does not). Voltages across capacitors cannot change instantaneously because this will result in infinite current flow.

c) (12 pts) Find the current and voltages after a very, very long time.

Provide the steps or explanation for your answers, e.g. using KCL/KVL, etc.

|       |        |
|-------|--------|
| $i_1$ | 1.11A  |
| $i_2$ | 0A     |
| $i_3$ | -0.11A |
| $i_4$ | 0A     |
| $v_1$ | 11.11V |
| $v_2$ | 25.56V |

3. (32 pts) Second-Order Circuit. **Remember to put down units.**



The switch is closed at  $t=0$ . The goal is to find the voltage across the capacitor,  $V(t)$ .

a.) (2 pts) For  $t < 0$ , assume that the switch was open and remained open for a very long time. Find  $V(0^-)$ .

$$V(0^-) = 3 \text{ V}$$

b.) (4 pts) What is  $V(0^+)$ ? Explain.

$$V(0^+) = 3 \text{ V.}$$

Because voltage across capacitor can not change abruptly.

c.) (10 pts) Derive the second-order differential equation for the circuit. (Hint: Use KVL/KCL)

Using KVL:

$$-3 + R(i_c(t) + i_L(t)) + V(t) = 0$$

$$-3 + R \left( C \frac{dV(t)}{dt} + \frac{1}{L} \int_{0^+}^t V(t') dt' + i_L(0^+) \right) + V(t) = 0$$

$$RC \frac{d^2V(t)}{dt^2} + \frac{R}{L} V(t) + \frac{dV(t)}{dt} = 0$$

or using KCL:

$$\frac{V(t) - 3}{R} + i_c(t) + i_L(t) = 0$$

$$\frac{V(t) - 3}{R} + C \frac{dV(t)}{dt} + \frac{1}{L} \int_{0^+}^t V(t') dt' + i_L(0^+) = 0$$

$$C \frac{d^2V(t)}{dt^2} + \frac{1}{L} V(t) + \frac{1}{R} \frac{dV(t)}{dt} = 0$$

In standard form:

$$\frac{d^2V(t)}{dt^2} + \frac{1}{RC} \frac{dV(t)}{dt} + \frac{1}{LC} V(t) = 0$$

$$\frac{d^2V(t)}{dt^2} + 6 \frac{dV(t)}{dt} + 9V(t) = 0$$

d.) (3 pts) What is  $\alpha$ ? What is  $\omega_0$ ? Is this critically damped, underdamped, or overdamped?

$$\alpha = 6/2 = 3 \text{ rad/s}$$

$$\omega_0 = \sqrt{9} = 3 \text{ rad/s}$$

$$\xi = \frac{\alpha}{\omega_0} = 1$$

Critically damped.

e.) (3 pts) What is the particular solution?

$v_p(t) = 0$  V since after a long time, the inductor acts as a short circuit.

f.) (10 pts) Find the complementary (homogeneous) solution.

$$v_c(t) = k_1 e^{-3t} + k_2 t e^{-3t}$$

$$v(t) = v_p(t) + v_c(t) = k_1 e^{-3t} + k_2 t e^{-3t}$$

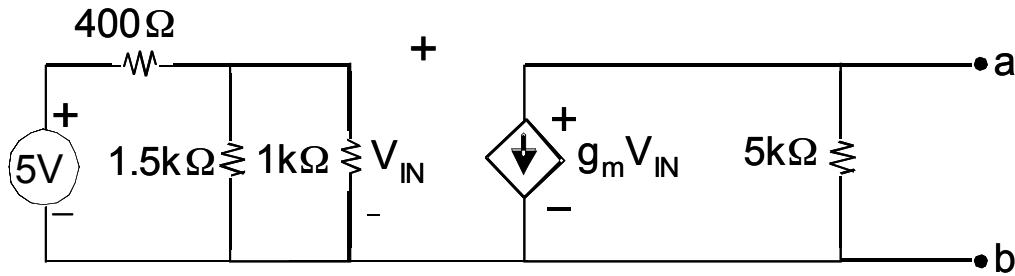
$$v(0) = 3 = k_1$$

$$dv(t)/dt|_{t=0} = 0 = -3k_1 + k_2 = -9 + k_2$$

$$k_2 = 9$$

$$v_c(t) = 3 e^{-3t} + 9 t e^{-3t}$$

4. (14 pts) Equivalent Circuit. **Remember to put down units.**



a.) (5 pts) What is  $V_{in}$ ?

$$1500 \times 1000 / (1500 + 1000) = 600 \Omega$$

$$V_{in} = 5 \times 600 / (600 + 400) = 3 \text{ V}$$

b.) (4 pts) What is  $V_{ab}$ ?

$$V_{ab} = -g_m V_{in} \times 5000 \text{ V}$$

$$= -g_m \times 3 \times 5000 \text{ V}$$

$$= -g_m \times 15000 \text{ V}$$

c.) (5 pts) What is the Thevenin equivalent circuit for terminals a-b? (Find  $R_{Th}, V_{Th}$ ).

Answer in terms of  $g_m$ .

$$V_{Th} = V_{ab} = -g_m \times 15000 \text{ V}$$

$$R_{Th} = V_{Th} / I_{sc} = -g_m \times 15000 / (-g_m \times 3) = 5000 \Omega$$