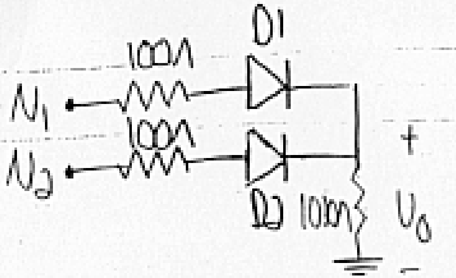
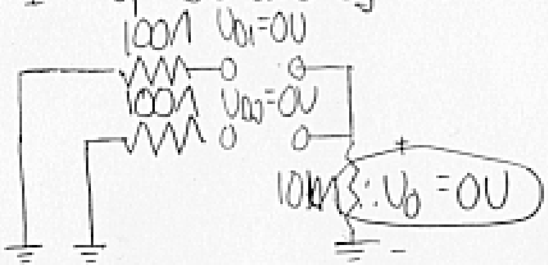
Midterm 1 Solution

1) 

V_1	V_2	V_{out}
0V (0)	0V (0)	0V (0)
0V (0)	5V (1)	~4.95V (1)
5V (1)	0V (0)	~4.95V (1)
5V (1)	5V (1)	~4.98V (1)

$V_{out} = V_1 + V_2$ (OR)

case 1: $V_1 = 0V$ and $V_2 = 0V$

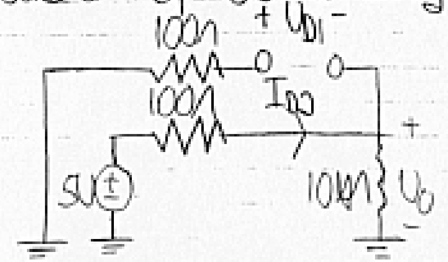


$\therefore V_0 = 0V$

assume that both D1 and D2 are off since there is no closed loop in the circuit there is no current in the circuit + which makes sense because there is no source in the circuit. note that $V_D \leq 0V$ for both diodes so our assumption that the diodes are off is correct

①

case 2: $V_1 = 0V$ and $V_2 = 5V$



$V_0 = 5V \frac{10k\Omega}{100\Omega + 10k\Omega}$

$V_0 = \frac{500}{101} V \approx 4.95V$

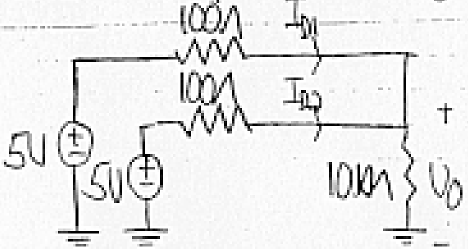
assume that D_1 is off and D_2 is on since $V_{D1} \approx -4.95V < 0V$ our assumption that D1 is off is correct. also since $I_{D2} > 0A$ our assumption that D2 is on is correct therefore the voltage divider equation shown above applies

case 3: $V_1 = 5V$ and $V_2 = 0V$

by symmetry, D1 is on, D2 is off, and $V_0 \approx 4.95V$

②

case 4: $V_1 = 5V$ and $V_2 = 5V$



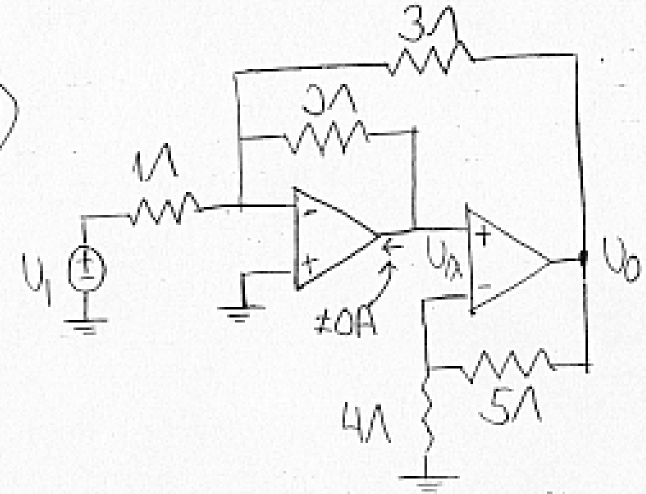
$$\frac{V_0 - 5V}{100\Omega} + \frac{V_0 - 5V}{100\Omega} + \frac{V_0}{10k\Omega} = 0A$$

$$\therefore V_0 = \frac{1000}{201} V \approx 4.975V$$

assume that both D1 and D2 are on
since both I_{D1} and I_{D2} > 0A our
assumption is correct and the
KCL equation above applies

③

2)



$$\frac{N_{01} - V_1}{1\Omega} + \frac{N_{01} - V_x}{2\Omega} + \frac{N_{01} - V_0}{3\Omega} = 0A \quad \text{(KCL at } N_{01}\text{)}$$

$$N_{01} = 0V$$

$$-\frac{V_1}{1\Omega} - \frac{V_x}{2\Omega} - \frac{V_0}{3\Omega} = 0A$$

$$6V_1 + 3V_x + 2V_0 = 0V$$

$$\frac{N_{02} - 0V}{4\Omega} + \frac{N_{02} - V_0}{5\Omega} = 0A \quad \text{(KCL at } N_{02}\text{)}$$

$$N_{02} = V_x$$

$$\frac{V_x}{4\Omega} + \frac{V_x}{5\Omega} - \frac{V_0}{5\Omega} = 0A$$

$$5V_x + 4V_x = 4V_0$$

$$V_x = \frac{4}{9} V_0$$

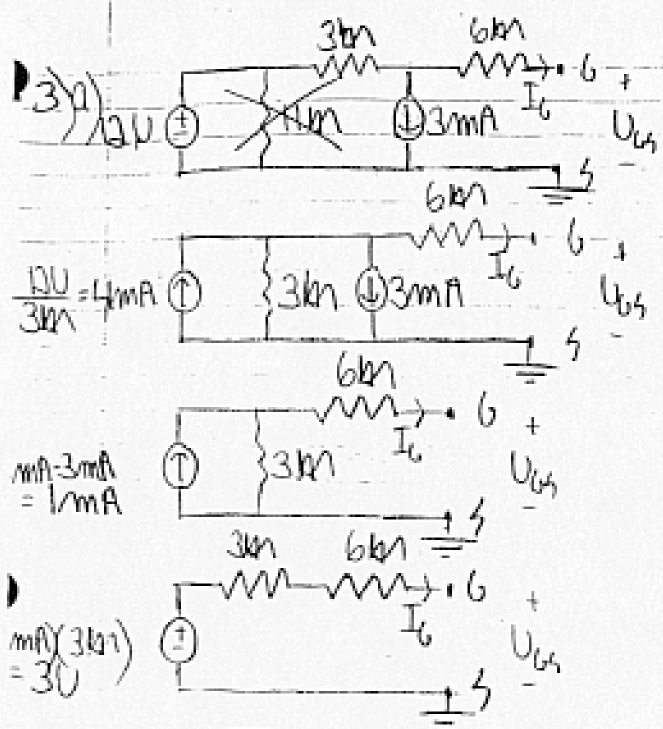
④

$$6V_1 + 3\left(\frac{4}{9}\cdot 3\,V_0\right) + 2V_0 = 0V$$

$$6V_1 + \frac{10}{3}V_0 = 0V$$

$$\therefore V_0 = -\frac{18}{10}V_1 = -\frac{9}{5}V_1 = -1.8V_1$$

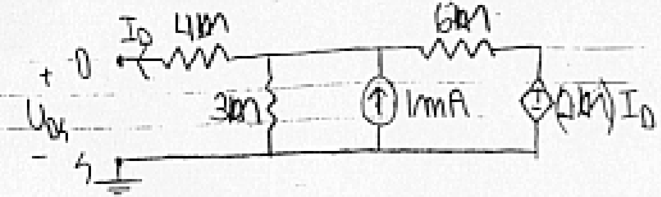
3) a)



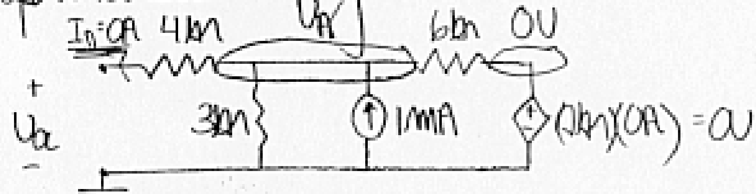
$\therefore V_{th} = 3V$

$R_{th} = 9k\Omega$

b)



open circuit voltage



nodal analysis

$$I_0 + I_{3k\Omega} + I_{1mA} + I_{6k\Omega} = 0A \quad (I_0 = 0A)$$

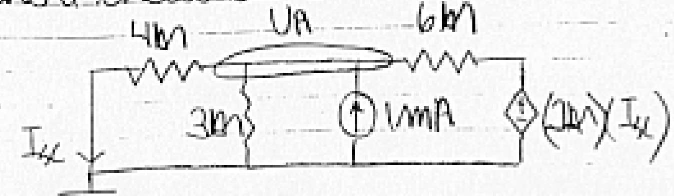
$$\frac{V_A - 0V}{3k\Omega} - 1mA + \frac{V_A - 0V}{6k\Omega} = 0mA$$

$$V_A = 2V$$

since $I_0 = 0A$

$$V_{oc} = V_A = 2V$$

short circuit current



nodal analysis

$$I_{sc} + I_{3k\Omega} + I_{1mA} + I_{6k\Omega} = 0A$$

$$\frac{V_A - 0V}{4k\Omega} + \frac{V_A - 0V}{3k\Omega} - 1mA + \frac{V_A - (2k)(I_{sc})}{6k\Omega} = 0mA$$

$$V_A\left(\frac{1}{4k\Omega} + \frac{1}{3k\Omega} + \frac{1}{6k\Omega}\right) - \frac{1}{3} I_{sc} = 1mA$$

$$I_{sc} = \frac{V_A - 0V}{4k\Omega} = \left(\frac{1}{4k\Omega}\right) V_A \Rightarrow \frac{1}{3} I_{sc} = \frac{1}{12k\Omega} V_A$$

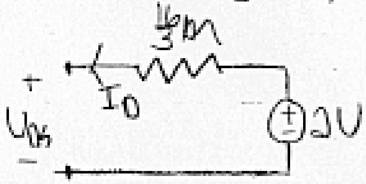
$$V_A\left(\frac{1}{4k\Omega} + \frac{1}{3k\Omega} + \frac{1}{6k\Omega} - \frac{1}{12k\Omega}\right) = 1mA$$

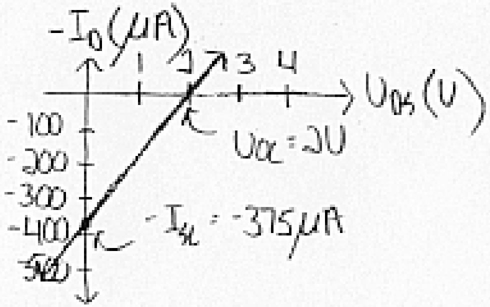
$$V_A = 1.5V$$

$$I_{sc} = \frac{V_A - 0V}{4k\Omega} = \frac{1.5V}{4k\Omega} = \frac{3}{8} mA$$

$V_{th} = V_{oc} = 2V \qquad I_N = I_{sc} = \frac{3}{8} mA = 375\mu A$

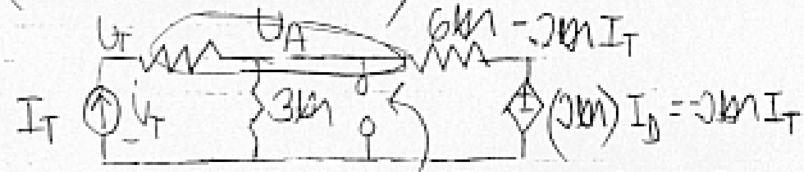
$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{2V}{(\frac{3}{8} mA)} = \frac{16}{3} k\Omega = 5.\bar{3} k\Omega$





⑨

alternative easier method to find R_0 (test source method)



kill current source

$-I_T + \frac{V_A}{3k\Omega} + \frac{V_A + 2k\Omega I_T}{6k\Omega} = 0A$ (KCL at V_T)

$-6k\Omega I_T + 2V_A + V_A + 2k\Omega I_T = 0V$

$3V_A - 4k\Omega I_T = 0V$

$V_T - I_T 4k\Omega = V_A$

$3(V_T - I_T 4k\Omega) - 4k\Omega I_T = 0V$

$3V_T - 16k\Omega I_T = 0V$

$\therefore R_{th} = R_{eq} = \frac{V_T}{I_T} = \frac{16}{3} k\Omega$

⑩

(6) c) What is the operating point of the MOSFET M1 (6 pts)? Put your answer in the box below. If you were unable to obtain answers for part a) and/or part b), then use the values shown below. Of course the values may or may not be correct:

part a) $V_{th} = 2V$, $R_{th} = 2k\Omega$

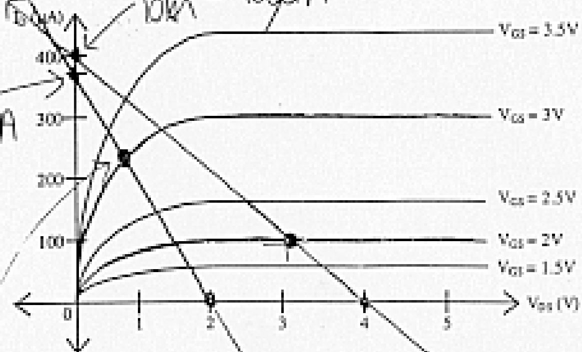
part b) $V_{th} = 4V$, $R_{th} = 10k\Omega$

Note: We will NOT cascade your errors or take off points for using the above values.

MOSFET M1 IV Curve

$\frac{4V}{10k\Omega} = 400\mu A$

$\frac{2V}{(16/3)k\Omega} = \frac{3}{8}mA = 375\mu A$



load line using correct values

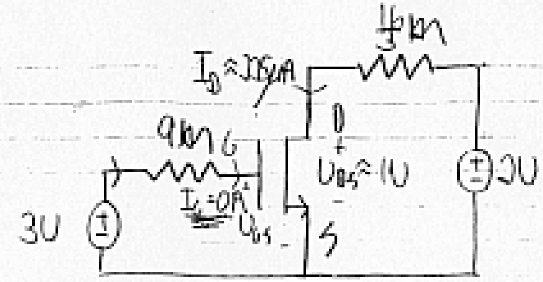
load line from given values

V_{DS} = 0.9V	I_D = 225μA

if given values used $V_{DS} \approx 3.1V$, $I_D \approx 100\mu A$

(11)

d)



correct values

$I_G = 0A$ $\sum P_{abs} = 0W$

$P_{3V} + P_{9k\Omega} + P_{\frac{4}{3}k\Omega} + P_{2V} + P_{m1} = 0W$ (0W, 0W)

$P_{m1} = |P_{2V} - P_{\frac{4}{3}k\Omega}|$ ($P_{m1(absorbed)} > 0W$)

$P_{m1} = |I_{2V}V_{2V} - I^2_{\frac{4}{3}k\Omega}R_{\frac{4}{3}k\Omega}|$

$P_{m1} \approx |(225\mu A)(2V) - (225\mu A)^2(\frac{16}{3}k\Omega)|$

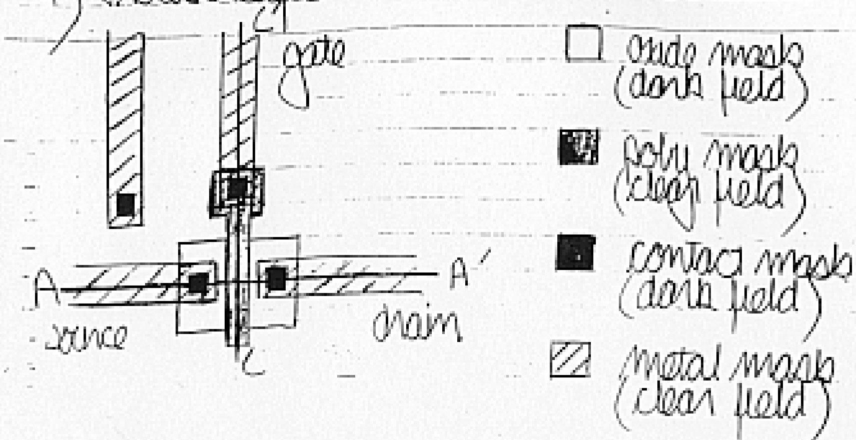
$\therefore P_{m1} \approx 180\mu W$

$P = I_D V_{DS} \approx (225\mu A)(0.8V) = 180\mu W$

many other possible final answers

(12)

4) Process sequence : NMOS transistor



starting material is a p-type silicon wafer

recipe

➢ ① grow 500 nm of SiO_2 (silicon dioxide) (field)

② pattern SiO_2 using the oxide mask (dark field)

➢ ③ grow 5 nm of SiO_2 (gate)

➢ ④ deposit 500 nm of n-type polysilicon

⑤ pattern polysilicon using the poly mask (clear field)

⑥ implant arsenic ($10^{19}\ cm^{-3}$) and anneal to form source and drain regions. note that the arsenic will penetrate the gate oxide but not the polysilicon or field oxide

⑦ deposit 500 nm of SiO_2

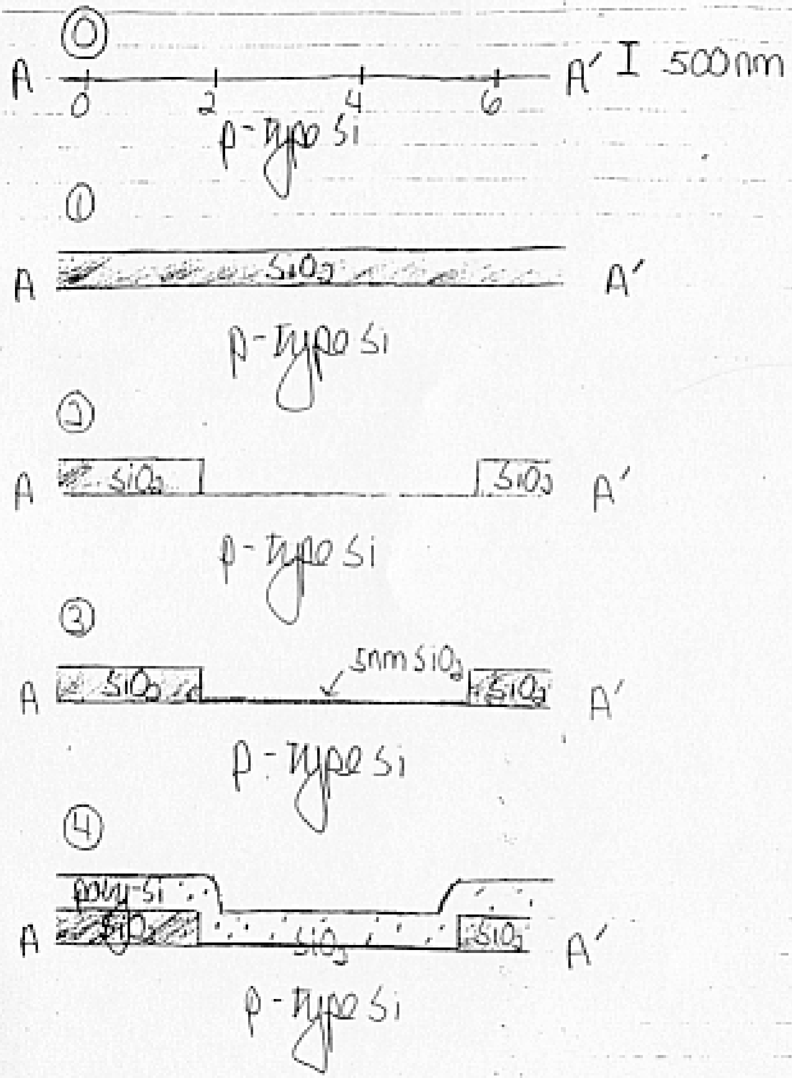
⑧ pattern SiO_2 using contact mask (dark field), etch long enough to remove all contacts

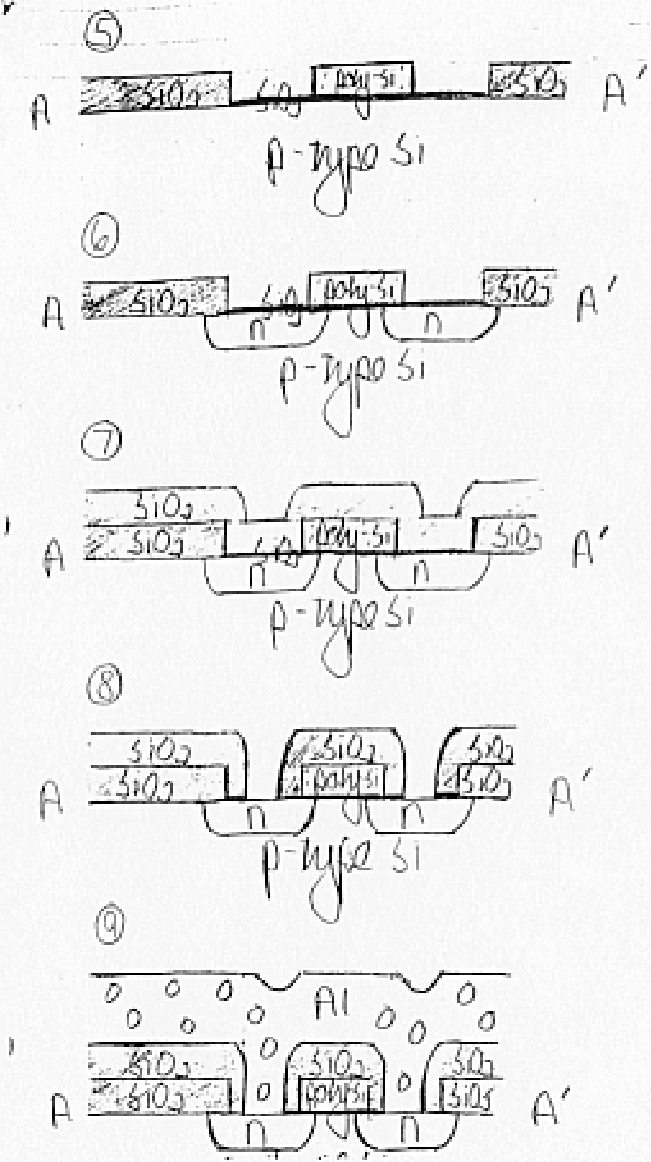
⑨ deposit 1 μm of Al

⑩ pattern Al using metal mask (clear field)

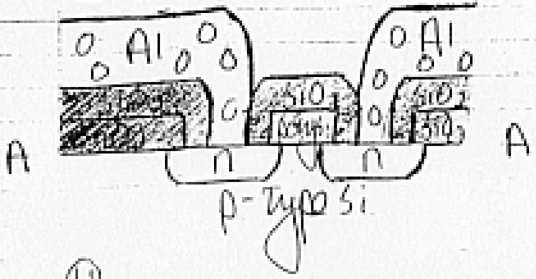
⑪ anneal to heal gate oxide damage and make good Si-Al contacts

AA' cross-section

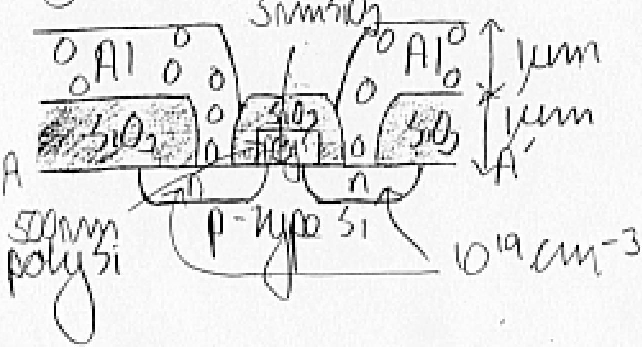




⑩


Al
Al
SiO_2
SiO_2
SiO_2
A
A'
n
n
p-type Si

⑪


5nm SiO_2
Al
Al
SiO_2
SiO_2
SiO_2
1µm
1µm
A
A'
n
n
500nm poly Si
p-type Si
10^{19} cm^{-3}