## EECS40, Spring 2000 <br> Midterm 1 solutions <br> Prof King

## Problem \#1


a)

| A | B | G |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

b)

## $G=(\operatorname{not} A)(n o t B)+A B$

c)

## 3 unit gate delays

The longest path between the input variables and the output variable is 3 logic gates. Therefore, we need to wait for a period of 3 unit gate delays after an input variable is changed, before we can trust the value of $G$ to be valid.
d)
Draw the timing diagrams for $t=0$ to $t=700 \mathrm{ps}$, for the given logic input values $A$ and $B$. $[10 \mathrm{pts}]$









| $E F$ | $G$ |  |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

$$
\begin{aligned}
& 1 \\
& 0
\end{aligned}
$$

logic value of $G=\overrightarrow{\varepsilon F}$

## Problem \#2

a)

## Rab = 13 ohms



## b)



* To achieve an equivalent resistance lower than the individual resistors, we should connect resistors in parallel
* But the parallel combination of 210 kohm resistors is 5 kohm -- too low!
=> need to increase the resistance of one of the parallel branches
* Try parallel combination of a 10 kohm resistor and 210 kohm resistors in series:
$(20 * 10) /(20+10)=6.7 \mathrm{kohm}-$ too high!
* Try increasing the resistance of one parallel branch by only 5 kohm ( 10 kohm II 10 kohm ) instead of 10 kohm
$\left(15^{*} 10\right) /(15+10)=6 \mathrm{kohm}!$


## c)

(ground is placed at the bottom of the diagram)

## i)

## Vcd $=4 \mathrm{~V}$

I3 $=0$ since terminal c is not connected. Thus the current flowing through R1 equals the current flowing through R2, i.e. we have a voltage divider.
$\Rightarrow \mathrm{Vbd}=(\mathrm{R} 2 /(\mathrm{R} 1+\mathrm{R} 2)) * 6=(2 /(1+2)) * 6=4 \mathrm{~V}$
Since there is no voltage drop across R 3 (because $\mathrm{I} 3=0$ ), $\mathrm{Vc}=\mathrm{Vb}$
$\Rightarrow>\mathrm{Vcd}=\mathrm{Vbd}=4 \mathrm{~V}$

## ii)

(underscore denotes subscript for uppercase variables)

## P_I = $\mathbf{6} \mathbf{~ m W}$ absorbed

The voltage across the current source is established by the voltage source and is equal to 6 V .
P_I $=\mathrm{IV}=(1 \mathrm{~mA})(6 \mathrm{~V})=6 \mathrm{~mW}$
Since positive current is entering the positive terminal of the current source it is absorbing power
iii)

| Parameter | value will: | Brief Explanation |
| :--- | :--- | :--- |
| Vbd | decrease | The resistance between b and d decreases; by the <br> voltage-divider formula, Vbd decreases |


| I1 | increase | Total resistance between a and d decreases; Vad remains <br> $6 \mathrm{~V} ; \mathrm{I} 1=\mathrm{Vad} /$ Rad |
| :--- | :--- | :--- |
| Power developed by voltage <br> source | increase | Since I1 increases, the current supplied by the voltage <br> source increases |

iv)

## $13=1.5 \mathrm{~mA}$

Equivalent resistance between terminals a and $d$ is
$\mathrm{R} 1+\mathrm{R} 2 \| \mathrm{R} 3=1+(2 * 2) /(2+2)=2 \mathrm{kohm}$
$=>\mathrm{I} 1=(6 \mathrm{~V}) /(2 \mathrm{kohm})=3 \mathrm{~mA}$
Using current-divider formula, $\mathrm{I} 3=(2 /(2+2))^{*}(3 \mathrm{~mA})=1.5 \mathrm{~mA}$

## Problem \#3

(underscore denotes subscript for uppercase variables)
a)

## nodal equations:

$(\mathrm{V}$ _AA -Va$) / \mathrm{R} 1+\mathrm{I}$ BB - I_CC $+(\mathrm{Vb}-\mathrm{Va}) / \mathrm{R} 3=0$
$(\mathrm{Va}-\mathrm{Vb}) / R 3-(\mathrm{V}-\bar{B} B+\mathrm{V} \overline{\mathrm{b}}) / \mathrm{R} 4+(\mathrm{Vc}-\mathrm{Vb}) / R 5=0$
I_CC + (Vb -Vc$) / \mathrm{R} 5-\mathrm{Vc} / \mathrm{R} 6=0$
Apply Kirchhoff's Current Law to nodes a, b, c:
(sum of currents entering a node $=0$ )
get 3 independent equations for 3 unknowns ( $\mathrm{Va}, \mathrm{Vb}, \mathrm{Vc}$ ) $\Rightarrow>$ can solve to find unknowns
b)

## nodal equations:

(V_AA - Va)/R1 + I_BB - (V_CC + Vb)/R3 + I_CC = 0
$(\mathrm{Vb}-\mathrm{Vc}) / \mathrm{R} 4+\mathrm{I} \mathbf{C} \bar{C}=\mathbf{0}$
$\mathbf{V b}-\mathbf{V a}=\mathbf{V}$ _BB


Current flowing through the voltage source V_BB cannot be expressed as a function of the node voltages Va and Vb
=> use the "supernode" approach
Applying Kirchhoff's Current Law to the supernode and node c:
supernode: $\left(\mathrm{V} \_\mathrm{AA}-\mathrm{Va}\right) / \mathrm{R} 1+\mathrm{I}$-BB $+\left(-\mathrm{V} \_\mathrm{CC}-\mathrm{Vb}\right) / \mathrm{R} 3+\mathrm{I}$-CC $=0$
node c : $(\mathrm{Vb}-\mathrm{Vc}) / \mathrm{R} 4+\mathrm{I}$ CC $=0$
Need one more equation in order to be able to solve for the 3 unknowns:
$\mathrm{Vb}-\mathrm{Va}=\mathrm{V} \_\mathrm{BB}$

## Problem \#4

a)

## V Th $=2 \mathrm{~V}$ <br> $\mathrm{R}_{-}^{-}$Th $=\mathbf{4}$ kohm

( x is the node between the 3 kohm and 2 kohm resistors)
The open-circuit voltage, Voc, is equal to Vab, which is equal to Vxb since no current is flowing through the 2 kohm resistor. Applying KCL to node x (defining node b as the reference node)
$->(5-V x) / 3+(-4-V x) / 6=0$
$\Rightarrow$ $6=3 \mathrm{Vx}=>\mathrm{Vx}=2 \mathrm{~V}$ therefore $\mathrm{Voc}=\mathrm{V}$ _Th $=2 \mathrm{~V}$
To find $R \_T h$, set all the independent sources to zero:

b)

## I N = 0.5 mA <br> $\overline{\mathrm{R}} \mathrm{N}=\mathbf{4} \mathrm{kohm}$

R_N = R_Th $=4$ kohm
$\mathrm{I} \_\mathrm{N}=\mathrm{V} \_\mathrm{Th} / \mathrm{R} \_\mathrm{Th}=(2 \mathrm{~V}) /(4 \mathrm{kohm})=0.5 \mathrm{~mA}$
c)


When $\mathrm{I}=0, \mathrm{~V}=-6 \mathrm{~V}$
When $\mathrm{V}=0$ (i.e. terminals a and b shorted together), $\mathrm{I}=(0-(-6 \mathrm{~V})) / 200=>\mathrm{I}=30 \mathrm{~mA}$

## d)

## P1k = 25 mW

nodal equations: $\left(\mathrm{V} \_\mathrm{AA}-\mathrm{Va}\right) / \mathrm{R} 1+\mathrm{I} \_\mathrm{BB}-\left(\mathrm{V} \_\mathrm{CC}+\mathrm{Vb}\right) / \mathrm{R} 3+\mathrm{I} \_\mathrm{CC}=0(\mathrm{Vb}-\mathrm{Vc}) / \mathrm{R} 4+\mathrm{I} \_\mathrm{CC}=0 \mathrm{Vb}-\mathrm{Va}=$ B_BB$_{-}$


Using voltage-divider formula,
$\mathrm{V}=(1000 /(1000+200)) *(-6)=-5 \mathrm{~V}$
$\mathrm{P}=\mathrm{IV}=(\mathrm{V} / \mathrm{R})^{*} \mathrm{~V}=\mathrm{V}^{\wedge} 2 / \mathrm{R}=\left((-5)^{\wedge} 2\right) / 1000=25 \mathrm{~mW}$

# Posted by HKN (Electrical Engineering and Computer Science Honor Society) <br> University of California at Berkeley <br> If you have any questions about these online exams <br> please contact examfile@hkn.eecs.berkeley.edu. 

