Fall 2011

EE 40

# Midterm 2

NAME: Solution

SSID:

*Instructions*

Read all of the instructions and all of the questions before beginning the exam.

There are 4 problems in this exam. The total score is 100 points. Points are given next to each problem to help you allocate time. Do not spend all your time on one problem.

Unless otherwise noted on a particular problem, you must show your work in the space provided, on the back of the exam pages or in the extra pages provided at the back of the exam. Simply providing answers will only result in partial credit, even if the answers are correct.

**Draw a BOX or a CIRCLE around your answers to each problem.**
Be sure to provide units where necessary.

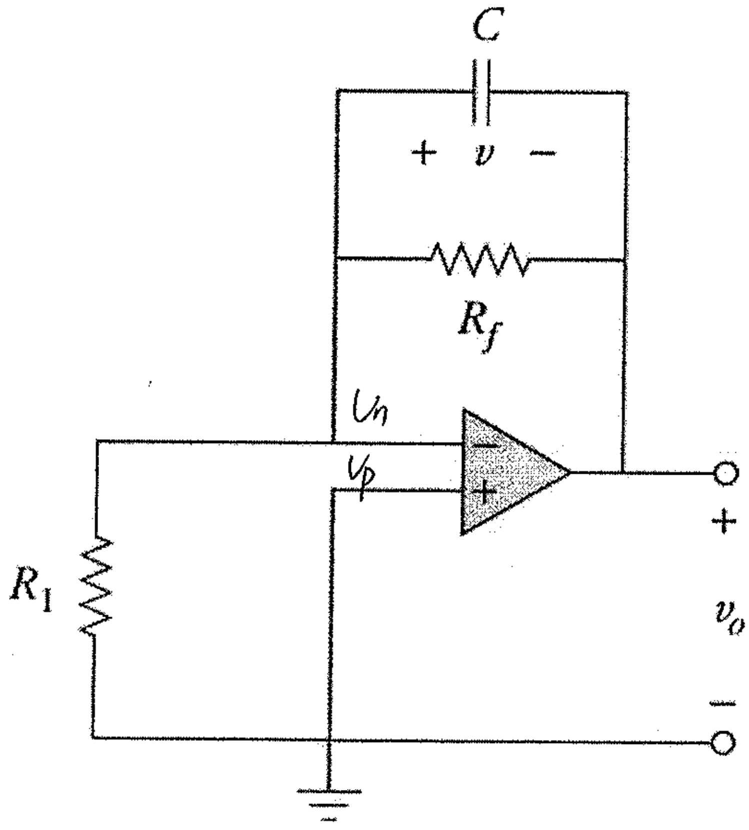
**GOOD LUCK!**

| PROBLEM | POINTS | MAX |
|---|---|---|
| 1 | | 15 |
| 2 | | 25 |
| 3 | | 40 |
| 4 | | 20 |

*"Pessimism is just an ugly word for pattern recognition."*
*-Anonymous*

**Problem 1** *Quickie* *(12.5 points)*

a) In the box below, provide a symbolic expression for $v_o$ for t > 0 if $v(0)$ = 4 V.




$$\boxed{V_o = -V(0)\, e^{-t/R_f C} \quad (V) \qquad t>0}$$

Ideal Omp : $V_n = V_p = 0$. ∴ No current goes into R1.

∴ $\tau = C \cdot R_f$.

$V_o(\infty) = 0$. , $V_o(0) = -V(0)$

∴ $V_o(t) = -V(0)\, e^{-t/R_f \cdot C}$ (V)

b) If $\mathbf{R_f}$ = 40 kΩ, $\mathbf{R_1}$ = 10 kΩ, **C** = 10 μF, and ***v(0)*** = 4 V, write the expression for $\boldsymbol{v_o}$ for t > 0 in the **BOX BELOW.** *(2.5 points)*

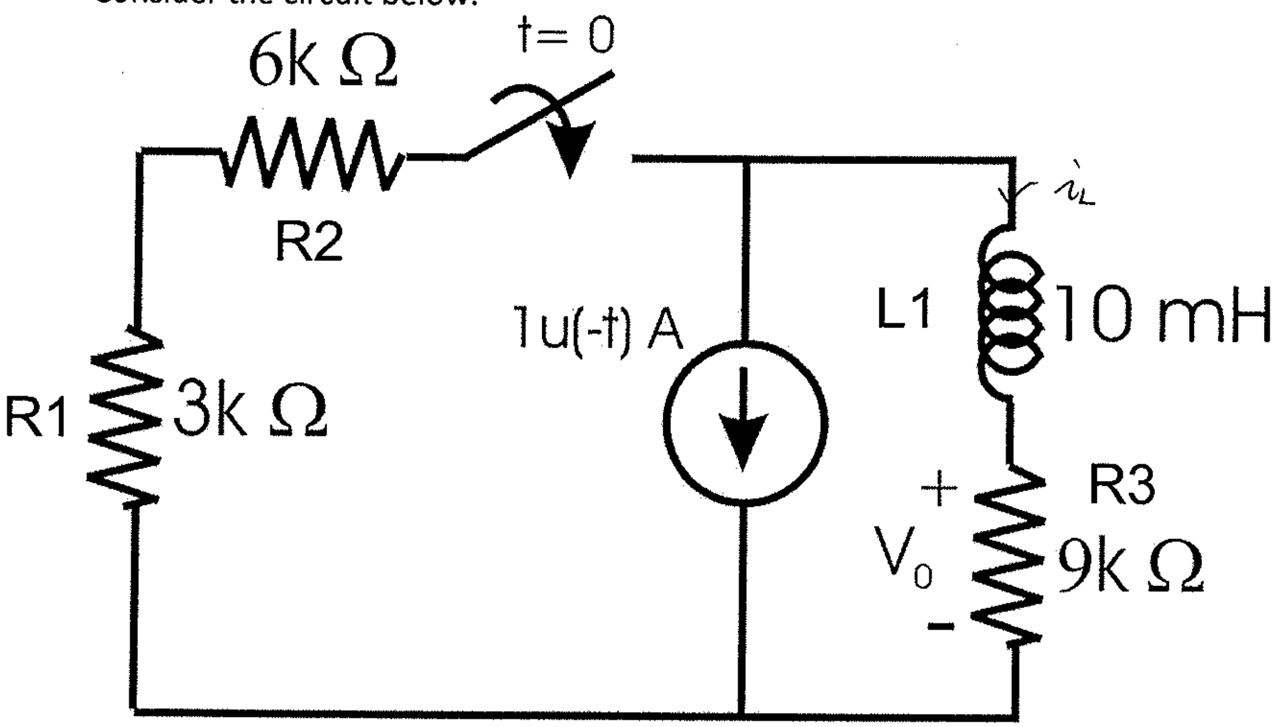
$$V_o(t) = -4e^{-t/0.4} \quad (V) \qquad t > 0.$$

$$R_f \cdot C = 40k \cdot 10\mu F = 0.4$$

*"If a person offends you... do not resort to extremes, simply watch your chance and hit him with a brick."*
*- Mark Twain*

**Problem 2** *First order circuits* *(25 points)*

Consider the circuit below.




a) What is the value of $V_0$ at $t = 0^+$? **Write It in the BOX BELOW.** *(5 points)*

$$V_o(0^+) = -9000\ V.$$

$$i_L(0^-) = -1A \Rightarrow i_L(0^+) = -1A.$$

$$V_o(0^+) = i_L(0^+) \cdot R_3 = -9000\ V.$$

b) *Using whatever method you like (yes, anything, don't raise your hand to ask if you can use XXX)*, provide a **symbolic expression** for the voltage $V_0(t)$ for $t > 0$ in the **BOX BELOW.** *(17.5 points)*

$$\boxed{V_0(t) = -R_3\, e^{-\frac{(R_1+R_2+R_3)t}{L}} \quad (V) \quad t > 0.}$$

$$\tau = \frac{L}{R_{eq}} = \frac{L_1}{R_1 + R_2 + R_3}$$

$$V(0^+) = -R_3$$

$$V(\infty) = 0.$$

$$V(t) = -R_3\, e^{-t/\tau}$$

c) Using the values provided in the figure, provide an expression for the voltage $V_0(t)$ for $t > 0$ in the **BOX BELOW.** *(2.5 points)*

$$V_0(t) = -9000\, e^{-1.8\times 10^6 t} \quad (V) \qquad t > 0.$$

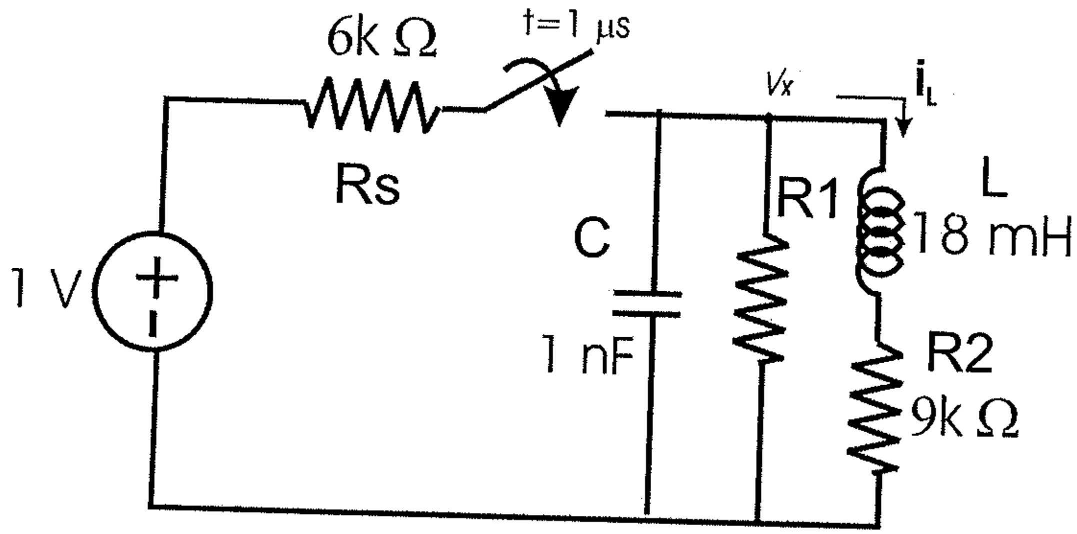
$R_3 = 9000.$

$$\frac{L_1}{R_1+R_2+R_3} = \frac{10\,mH}{3k+6k+9k} = \frac{5}{9}\times 10^{-6}$$

*"Accept that some days you are the pigeon, and some days you are the statue."*
*-David Brent, The Office*

**Problem 3** *Second order circuits* *(40 points)*

Consider the circuit below.




a) Find the second order **differential equation** for the variable $i_L$ that describes the circuit behavior for t > 1 µs. Write into the box below. *(15 points)*

$$\frac{d^2 i_L}{dt^2} + \left(\frac{1}{R_sC} + \frac{1}{R_1C} + \frac{R_2}{L}\right)\frac{di_L}{dt} + \frac{1}{LC}\left(\frac{R_2}{R_s} + \frac{R_2}{R_1} + 1\right) i_L = \frac{1}{LCR_s}$$

KCL @ Vx $\quad \frac{V_x - 1}{R_s} + C\frac{dV_x}{dt} + \frac{V_x}{R_1} + i_L = 0 \quad$ ①

$$V_x = L\frac{di_L}{dt} + i_L R_2 \quad ②$$

② → ① $\quad \frac{L}{R_s}\frac{di_L}{dt} - \frac{1}{R_s} + \frac{R_2}{R_s} i_L + CL\frac{d^2 i_L}{dt^2} + CR_2\frac{di_L}{dt} + \frac{L}{R_1}\frac{di_L}{dt} + \frac{R_2}{R_1} i_L + i_L = 0$

$\Rightarrow \quad CL\frac{d^2 i_L}{dt^2} + \left(\frac{L}{R_s} + CR_2 + \frac{L}{R_1}\right)\frac{di_L}{dt} + \left(\frac{R_2}{R_s} + \frac{R_2}{R_1} + 1\right) i_L = \frac{1}{R_s}$

$\Rightarrow \quad \frac{d^2 i_L}{dt^2} + \left(\frac{1}{R_sC} + \frac{R_2}{L} + \frac{1}{R_1C}\right)\frac{di_L}{dt} + \frac{1}{LC}\left(\frac{R_2}{R_s} + \frac{R_2}{R_1} + 1\right) i_L = \frac{1}{LCR_s}$

b) Is there a value for R1 such that the resonant circuit can be ***critically damped*** for t > 1 μs? *(5 points)*

*Please show your equations clearly.*

| **If YES, write the value here:** | **If NO, provide an expression that shows why not here:** |
|---|---|
| $R_1 = -3 + 3\sqrt{2}\ \ k\Omega$ | |

$$\alpha = \frac{1}{2}\left(\frac{R_2}{L} + \frac{1}{CR_S} + \frac{1}{CR_1}\right) = \frac{1}{2}\left(\frac{9K}{18mH} + \frac{1}{1nF\times 6K} + \frac{1}{1nF\times R_1}\right) = \left(\frac{1}{3} + \frac{1}{2R_1}\right)\times 10^6$$

$$w_0 = \sqrt{\frac{1}{LC}\left(\frac{R_2}{R_S} + \frac{R_2}{R_1} + 1\right)} = \sqrt{\frac{1}{1nF\times 18mH}\left(\frac{9}{6} + \frac{9}{R_1} + 1\right)} = \sqrt{\frac{1}{2R_1} + \frac{5}{36}}\ \times 10^6$$

Critical damped $\alpha = w_0$

$$\therefore \left(\frac{1}{3} + \frac{1}{2R_1}\right)\times 10^6 = \sqrt{\frac{1}{2R_1} + \frac{5}{36}}\ \times 10^6$$

$$\Rightarrow \frac{1}{9} + \frac{1}{3R_1} + \frac{1}{4R_1^2} = \frac{1}{2R_1} + \frac{5}{36}$$

$$\Rightarrow 4R^2 + 12R_1 + 9 = 18R_1 + 5R_1^2$$

$$\Rightarrow R_1^2 + 6R_1 - 9 = 0$$

$$\Rightarrow R_1 = \frac{-6 + \sqrt{36+36}}{2} = -3 + 3\sqrt{2}\ \ k\Omega.$$

c) Determine the **two** *relevant* initial conditions of the circuit *(5 points)*

| **Condition 1:** |
|---|
| $i_L(1\mu s^+) = 0$ |
| **Condition 2:** |
| $i_L'(1\mu s^+) = 0$ |

$i_L(1\mu s^-) = 0 \quad \Rightarrow \quad i_L(1\mu s^+) = 0$

$V_C(1\mu s^-) = 0 \quad \Rightarrow \quad V_C(1\mu s^+) = 0$

$L\, i_L'(1\mu s^+) + i_L(1\mu s^+) \cdot R_2 = V_C(1\mu s^+)$

$\Rightarrow i_L'(1\mu s^+) = 0$

d) **Assuming R1 = 9000 Ω**, provide a complete expression for $i_L(t)$ for $t > 1\ \mu s$.

*(15 points)*

$$i_L(t) = \frac{1}{21} + e^{-\frac{7}{18}\times 10^6 (t-1\mu s)}\left[\frac{-1}{21}\cos\left(\frac{\sqrt{14}}{18}\times 10^6 (t-1\mu s)\right) - \frac{1}{3\sqrt{14}}\sin\left(\frac{\sqrt{14}}{18}\times 10^6 (t-1\mu s)\right)\right]\ mA$$

From b.

$$\alpha = \left(\frac{1}{3} + \frac{1}{2R_1}\right)\times 10^6 = \frac{7}{18}\times 10^6$$

$$\omega_0 = \sqrt{\frac{5}{36} + \frac{1}{2R_1}}\times 10^6 = \sqrt{\frac{7}{36}}\times 10^6$$

$$\alpha^2 - \omega_0^2 = \left[\frac{7\times 7}{18\times 18} - \frac{7\times 9}{36\times 9}\right]\times 10^{12} < 0 \quad \therefore \alpha < \omega_0$$

Underdamped.

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{\frac{(63-49)}{18\times 18}\times 10^{12}} = \frac{\sqrt{14}}{18}\times 10^6$$

$$i_L(\infty) = \frac{1}{2}\cdot\frac{1}{6k + (9k // 9k)} = \frac{1}{21k} = \frac{1}{21}\ mA.$$

$$i_L(t) = i_L(\infty) + e^{-\alpha(t-1\mu)}\left[B_1 \cos\omega_d(t-1\mu) + B_2 \sin\omega_d(t-1\mu)\right]$$

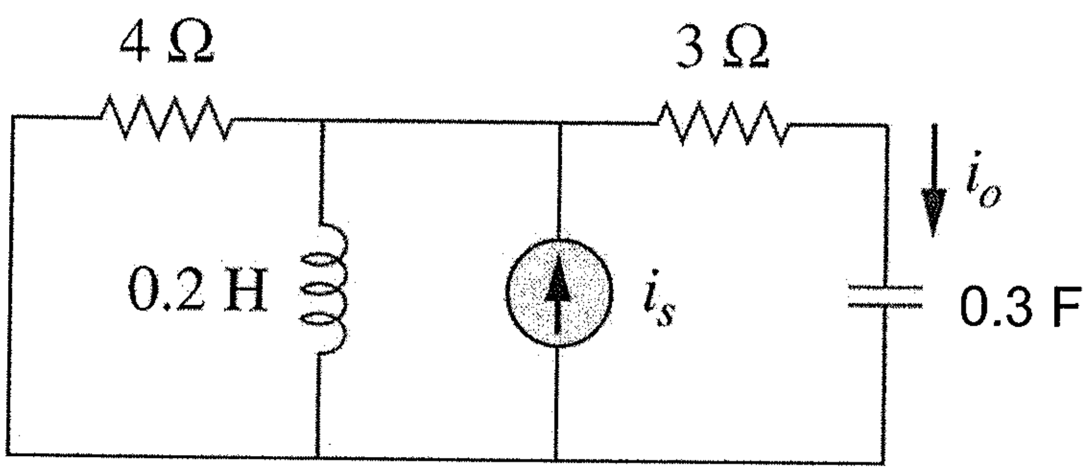
$$B_1 = -i_L(\infty) = \frac{-1}{21}\times 10^{-3}$$

$$B_2 = \frac{i_L'(0) + \alpha\left[i_L(0) - i_L(\infty)\right]}{\omega_d} = \frac{-\frac{7}{18}\times 10^6 \times \frac{1}{3\cdot 21}\times 10^{-3}}{\frac{\sqrt{14}}{18}\times 10^6} = \frac{-1}{3\sqrt{14}}\times 10^{-3}$$

*"I know they were just kids...but man we beat the f$%! out of them!"*
*- Dogma*

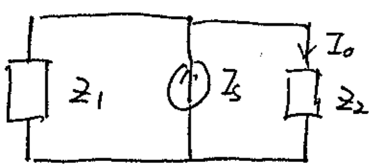
**Problem 4** *Phasors* (20 points)

If $i_s = 5\cos(10t + 40°)$ A in the circuit below, find $i_0$




$$i_o(t) = \frac{30\sqrt{2}}{19}\cos(10t + 85°)\ A$$

$$I_s = 5e^{j40°}$$




$$Z_1 = 4 \parallel 2j = \frac{8j}{4+2j}$$

$$Z_2 = 3 + \frac{1}{3j}$$

$$I_o = \frac{Z_1}{Z_1 + Z_2} \cdot I_s$$

$$I_o = \frac{\frac{8j}{4+2j}}{\frac{8j}{4+2j} + 3 + \frac{1}{3j}} \cdot 5e^{j40°} = \frac{8j \times 5e^{j40°}}{8j + 12 + 6j - \frac{4}{3}j + \frac{2}{3}} = \frac{40e^{j130°}}{\frac{38}{3} + \frac{38}{3}j}$$

$$= \frac{40}{\frac{38}{3}\sqrt{2}} \cdot \frac{e^{j130°}}{e^{j45°}} = \frac{30}{19}\sqrt{2}\, e^{j85°}$$

*Page for extra work*

| x | atan(x) radians | atan(x) degrees |
|---|---|---|
| 0 | 0 | 0 |
| 0.1 | 0.099668652 | 5.710593137 |
| 0.2 | 0.19739556 | 11.30993247 |
| 0.3 | 0.291456794 | 16.69924423 |
| 0.4 | 0.380506377 | 21.80140949 |
| 0.5 | 0.463647609 | 26.56505118 |
| 1 | 0.785398163 | 45 |
| 1.5 | 0.982793723 | 56.30993247 |
| 2 | 1.107148718 | 63.43494882 |
| 3 | 1.249045772 | 71.56505118 |
| 4 | 1.325817664 | 75.96375653 |
| 5 | 1.373400767 | 78.69006753 |
| 6 | 1.405647649 | 80.53767779 |
| 100 | 1.56079666 | 89.4270613 |