UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering and Computer Sciences

EECS 40
Introduction to Microelectronic Circuits

Fall 2003

MIDTERM EXAMINATION #1
September 29, 2003
Time allotted: 50 minutes

NAME: SOLUTIONS , ____________
(print) Last, First

Signature: ____________ **STUDENT ID#:** ____________

Discussion Section: ____________

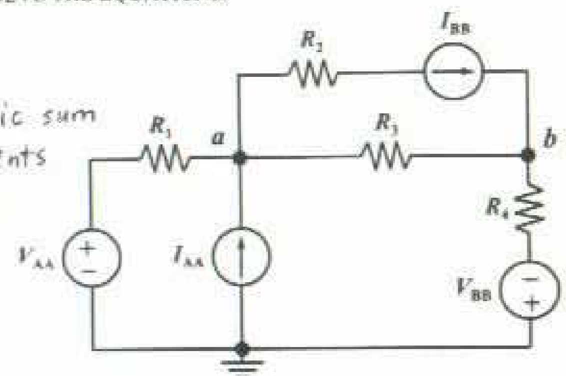
1. This is a **CLOSED BOOK** exam. However, you may use 1 page of notes and a calculator.
2. **SHOW YOUR WORK** on this exam. **(Make your methods clear to the grader.)**
3. **Write your answers clearly in the spaces (lines, boxes, or plots) provided.**
4. Remember to specify the units on answers whenever appropriate.

SCORE:	1 ______	/ 15
	2 ______	/ 21
	3 ______	/ 14
Total:	______	/ 50

Problem 1: Circuit Analysis [15 points in total]

a) In the circuit below, the independent source values and resistances are known. Use the node-voltage method to write 2 equations sufficient to solve for V_a and V_b. To receive credit, you must write your answer in the box below. **[5 pts]**
DO NOT SOLVE THE EQUATIONS!

KCL:
The algebraic sum of all currents flowing into a node is zero.



Applying KCL to node a:

$$\frac{V_{AA}-V_a}{R_1} + I_{AA} - I_{BB} + \frac{V_b - V_a}{R_3} = 0$$

Applying KCL to node b:

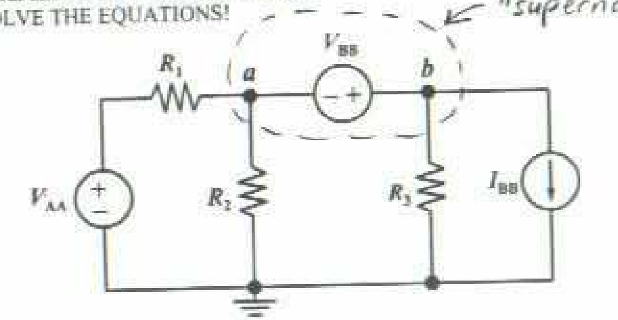
$$\frac{V_a - V_b}{R_3} + I_{BB} + \frac{-V_{BB}-V_b}{R_4} = 0$$

Write your equations here:

$$\frac{V_{AA}-V_a}{R_1} + I_{AA} - I_{BB} + \frac{V_b - V_a}{R_3} = 0$$

$$\frac{V_a - V_b}{R_3} + I_{BB} - \frac{V_{BB}+V_b}{R_4} = 0$$

b) In the circuit below, the independent source values and resistances are known. Use the node-voltage method to write 2 equations sufficient to solve for V_a and V_b. To receive credit, you must write your answer in the box below. **[5 pts]**
DO NOT SOLVE THE EQUATIONS!



Applying KCL to supernode:

$$\frac{V_{AA}-V_a}{R_1} + \frac{0-V_a}{R_2} + \frac{0-V_b}{R_3} - I_{BB} = 0$$

Constraint imposed by floating voltage source:

$$V_b - V_a = V_{BB}$$

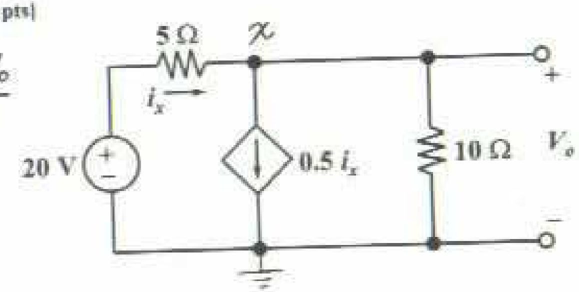
Write your equations here:

$$\frac{V_{AA}-V_a}{R_1} - \frac{V_a}{R_2} - \frac{V_b}{R_3} - I_{BB} = 0$$

$$V_b - V_a = V_{BB}$$

c) Find V_o. **[5 pts]**

$$i_x = \frac{20-V_o}{5}$$



Applying KCL to node x:

$$i_x - 0.5\,i_x - \frac{V_o - 0}{10} = 0$$

$$0.5\,i_x - \frac{V_o}{10} = 0$$

$$i_x - \frac{V_o}{5} = 0$$

$$\frac{20-V_o}{5} - \frac{V_o}{5} = 0$$

$$20 - V_o - V_o = 0$$

$$20 = 2V_o$$

$$10 = V_o$$

V_o = 10 Volts

Problem 2: Equivalent Circuits [21 points in total]

a) Suppose you are given five resistors, each of value 10 kΩ.

i) What is the **maximum resistance** which can be achieved by connecting these five resistors? Show how they should be connected in this case. **[3 pts]**

Connect the resistors in series, in order to achieve the largest equivalent resistance. $R_{eq} = \sum_{i=1}^{5} R_i = 50\ k\Omega$

Circuit diagram of resistors connected to give a **maximum resistance value** of 50 kΩ

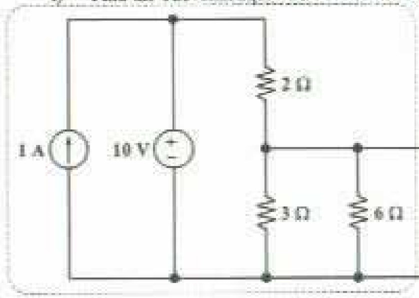
ii) What is the **minimum resistance** which can be achieved by connecting these five resistors? Show how they should be connected in this case. **[3 pts]**

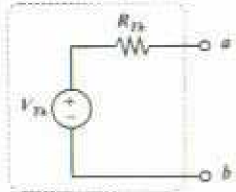
Connect the resistors in parallel, in order to achieve the smallest equivalent resistance.

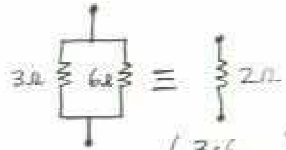
$$\frac{1}{R_{eq}} = \sum_{i=1}^{5} \frac{1}{R_i} = 5\left(\frac{1}{10k\Omega}\right) = \frac{1}{2k\Omega}$$

Circuit diagram of resistors connected to give a **minimum resistance value** of 2 kΩ

i) Find the Thevenin equivalent circuit for the circuit below. **[6 pts]**

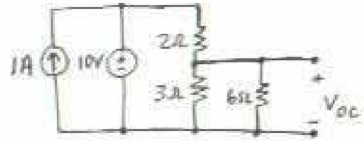




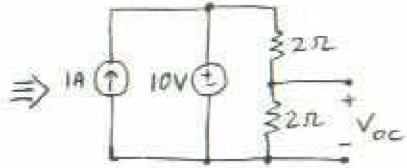


$$\left(\frac{3\cdot 6}{3+6} = 2\right)$$

V_{Th} is just the open-circuit voltage V_{oc}:



$\Rightarrow$

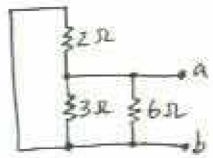


Using the voltage-divider formula: $V_{oc} = \frac{2\Omega}{2\Omega + 2\Omega}(10V)$

$V_{oc} = \underline{\underline{5V}}$

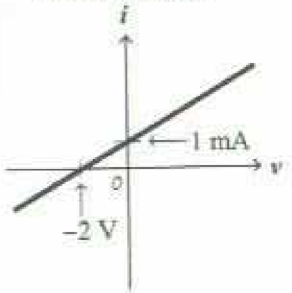
To find R_{Th}, set all the independent sources to zero:

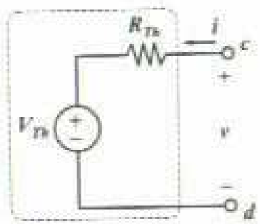
- current source → open circuit
- voltage source → short circuit



V_{Th} =	5 Volts
R_{Th} =	1 Ω

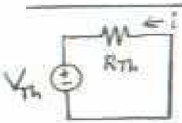
ii) The *I-V* characteristic of a linear circuit is given below. Find the Thévenin equivalent of this circuit. **[3 pts]**





When $i = 0$, $v = V_{Th}$: From the I-V plot, $v = -2V$ when $i = 0$. Therefore, $V_{Th} = -2V$

When $v = 0$, $i = -\frac{V_{Th}}{R_{Th}}$:



From the I-V plot, $i = 1mA$ when $v = 0$.

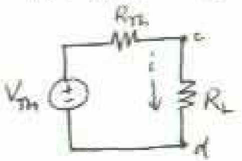
$1\,mA = -\frac{(-2V)}{R_{Th}}$

V_{Th} = −2 Volts

R_{Th} = 2 kΩ

iii) Suppose the circuit in part **(c)** is loaded with a resistor of resistance R_L (connected between terminals *c* and *d*). What is the maximum power that can be delivered to this load resistor? **[3 pts]** (Hint: You should choose the value of R_L which results in the maximum power absorbed by the load resistor, and then calculate that power.)

From the maximum power transfer theorem, $R_L = R_{Th}$ for maximum power to be absorbed by R_L.



P_{max} = 0.5 mW

Power delivered to load resistor is $i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L = \frac{V_{Th}^2}{4R_{Th}} = \frac{(-2)^2}{4(2k\Omega)} = \frac{1}{2}\,mW$

Problem 3: Op Amp Circuit [14 points in total]

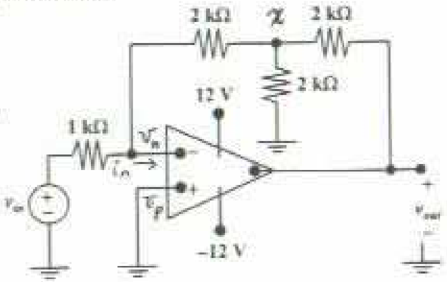
Consider the op amp circuit below:

ideal op amp:

$v_n = v_p = 0\,V$

$i_n = 0\,A$

$i_p = 0\,A$



a) Assuming the op amp is operating in its linear region, find an expression for v_{out} (as a function of v_{in}). **[10 pts]**

Applying KCL to inverting input node: $\frac{v_{in}-0}{1k\Omega} + \frac{V_x - 0}{2k\Omega} = 0$

$\Rightarrow V_x = -2v_{in}$

Applying KCL to node x:

$\frac{0-V_x}{2k\Omega} + \frac{0-V_x}{2k\Omega} + \frac{v_{out}-V_x}{2k\Omega} = 0 \quad \Rightarrow v_{out} = 3V_x = 3(-2v_{in})$

v_o = $-6v_{in}$

b) Sketch the voltage transfer characteristic for the op-amp circuit, for v_{in} ranging from −5 Volts to +5 Volts. Indicate the minimum and maximum values of v_{out}. **[4 pts]**

Due to the positive and negative power supply voltages, $-12V \le v_{out} \le 12V$