EECS 20N: Structure and Interpretation of Signals and Systems   MIDTERM 1

Department of Electrical Engineering and Computer Sciences   14 February 2006

UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name _____ FIRST Name _____

Lab Time _____

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.

- This examshould take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.

- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except one double-sided 8.5" × 11" sheet of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff— *including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.

- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.

- Please write neatly and legibly, because *if we can't read it, we can't grade it.*

- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*

- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.

- We hope you do a *fantastic* job on this exam.

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MT1.1 (25 Points) Every inhabitant of the planet Zirth is called a Zirthling. Data collected by our ZirthRover has revealed the existence of the following non-empty categories (subsets) of living Zirthlings:

- F: A Zirthling belonging to this set is called a Fubricon.
- G: A Zirthling belonging to this set is called a Gubricon.
- H: A Zirthling belonging to this set is called a Hubricon.

We know the following about the Zirthlings:

(i) Some Fubricons are Gubricons (i.e., $\exists\, z \in \mathsf{F}$ such that $z \in \mathsf{G}$).

(ii) A Zirthling can*not* be a Fubricon unless it is also a Hubricon.

Consider the assertion
$$\{z \in \mathsf{H} | z \in \mathsf{G}\} = \phi\,,$$
where ϕ denotes the empty set.

From the choices below, select the strongest correct statement about the assertion above. Explain your reasoning succinctly, but clearly and convincingly.

(I) The assertion must be true.

(II) The truth or falsehood of the assertion cannot be determined based on the information given.

(III) The assertion must be false.

MT1.2 (25 Points) Consider the three sets A, B, and C depicted below:

$$A = \{a\}, \quad B = \{a, b\}, \quad C = \{a, b, c\}.$$

Determine the truth or falsehood of each of the assertions below. If you find an assertion to be false, provide a counterexample by finding an element in the purported subset which is not a member of the purported superset. If you find an assertion to be true, prove that every element in the purported subset is a member of the purported superset.

(i) $[A \rightarrow C] \subset [B \rightarrow C]$.

    Circle one: TRUE   FALSE   INDETERMINATE

    Explanation:

(ii) $[A \rightarrow B] \subset [A \rightarrow C]$.

    Circle one: TRUE   FALSE   INDETERMINATE

    Explanation:

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MT1.3 (25 points) Consider a continuous-time (CT) system

$$F : [\mathbb{R} \to \mathbb{R}] \to [\mathbb{R} \to \{-1, 0, +1\}],$$

which acts as a crude quantizer, described below. The output signal y, produced by the system F in response to an appropriately-defined (but otherwise arbitrary) input signal x, is characterized as follows:

$$\forall t \in \mathbb{R}, \quad y(t) \triangleq \begin{cases} -1 & \text{if } x(t) < 0 \\ 0 & \text{if } x(t) = 0 \\ +1 & \text{if } x(t) > 0. \end{cases}$$

For each part (a)-(d), select the strongest true assertion from the list. Provide a succinct, but clear and convincing, explanation for each of your selections. If your selection in any part is "(iii)", i.e., that the system can*not* have the particular property in question, you must provide a counterexample.

(a) MEMORYLESSNESS

   (i) The system must be memoryless, because there exists a function f such that $y(t) = f(x(t))$, $\forall t \in \mathbb{R}$, and $\forall x \in [\mathbb{R} \to \mathbb{R}]$. If this is your selection, specify the function f.

  (ii) The system could be memoryless, but does not have to be.

 (iii) The system cannot be memoryless.

(b) CAUSALITY

  (i) The system must be causal.

  (ii) The system could be causal, but does not have to be.

  (iii) The system cannot be causal.

(c) TIME INVARIANCE

  (i) The system must be time-invariant.

  (ii) The system could be time-invariant, but does not have to be.

  (iii) The system cannot be time-invariant.

(d) LINEARITY

  (i) The system must be linear.

  (ii) The system could be linear, but does not have to be.

  (iii) The system cannot be linear.

MT1.4 (30 points) Consider a discrete-time (DT) system

$$G : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}].$$

The DT unit impulse signal (i.e., the Kronecker delta function)

$$\delta : \mathbb{Z} \to \mathbb{R}$$
$$\forall\, n \in \mathbb{Z}, \quad \delta(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases}$$

is applied as input to the system, in response to which the system produces the output signal

$$y : \mathbb{Z} \to \mathbb{R}$$
$$\forall\, n \in \mathbb{Z}, \quad y(n) = \begin{cases} \alpha & \text{if } n < 0 \\ 2 & \text{if } n = 0 \\ \beta & \text{if } n > 0, \end{cases}$$

where α and β are real constants.

(a) For this part only, assume that the system G is memoryless and $\alpha = \pi$. Then there must exist a function $f : \mathbb{R} \to \mathbb{R}$ such that

$$y(n) = f(x(n)),$$

for every $n \in \mathbb{Z}$ and for every input signal $x \in [\mathbb{Z} \to \mathbb{R}]$.

(i) Determine f to the extent possible (you will not be able to specify f completely). Note that as part of determining f, you must also determine β.

(ii) Can the system G (of this part (a)) be linear? Explain your reasoning succinctly, but clearly and convincingly.

(b) For this part only, assume that the system G is linear and memoryless. Determine α and β.

(c) For this part only, assume that the system G is linear and causal (but not necessarily memoryless). What constraint(s), if any, must α and β satisfy if the system G is to be both linear and causal?

LAST Name ＿＿＿＿＿＿＿＿＿＿＿ FIRST Name ＿＿＿＿＿＿＿＿＿＿＿

Lab Time ＿＿＿＿＿＿＿＿＿＿＿

Problem	Points	Your Score
Name	10	
1	25	
2	25	
3	25	
4	30	
Total	**115**	

Do not write above this line. You may use the blank space below for scratch work. Nothing written on this page will be considered in evaluating your work.