UNIVERSITY OF CALIFORNIA

College of Engineering

Department of Electrical Engineering and Computer Sciences

Professor Tse | Spring 2005

EECS 20n — Midterm 2 Solutions

[24 pts.] Question 1

For a) below, answer Yes or No. No partial credit.

[6] a) (i) Is the system:

$$\ddot{y}(t) + ay(t) = bx(t), \qquad t \geq 0$$
$$y(0) = 0$$
$$\dot{y}(0) = 1$$

linear? **Answer:** No

time invariant? **Answer:** No

memoryless? **Answer:** No

[6] (ii) Is the system:

$$y(n+1) = y(n) + 3x(n) + 2x(1), \qquad n \geq 0$$
$$y(0) = 0$$

linear? **Answer:** Yes

time invariant? **Answer:** No

memoryless? **Answer:** No

[6] (iii) Is the system:

$$y(t) = x^2(t) - x^2(t-1)$$

linear? **Answer:** No

time invariant? **Answer:** Yes

memoryless? **Answer:** No

[6] b) Consider a discrete-time linear system S with inputs $x = \text{Integers} \to \text{Reals}$. Suppose you are given the outputs of the system corresponding to the inputs $x_k(n) = \delta(n-k)$, for all integers k. Is that enough information for you to find the output of S for an arbitrary input x? Explain.

- Yes. (+2 pts.)
- Inputs can be expressed as a *linear combination* of $x_k(n)$. (+2 pts.)
- Use *linearity* of system to determine outputs. (+2)

[18 pts.] Question 2

For each of the following discrete-time LTI systems with given impulse responses, can it be realized by a linear state machine, i.e., can you find $(A, \vec{b}, \vec{c}, d)$ such that

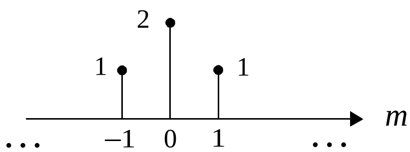
$$\vec{s}(n+1) = A\,\vec{s}(n) + \vec{b}x(n)$$
$$y(n) = \vec{c}^{\,\tau}\,\vec{s}(n) + dx(n)$$

such that

$$y = h * x\ ?$$

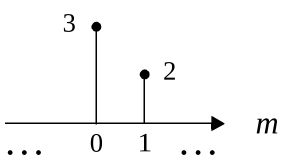
If so, give one. If not, explain.

[6] (a)



No (+2 pts.)
Acausal system and $h(-1)$ cannot be zero.
(+4 pts.)

[6] (b)



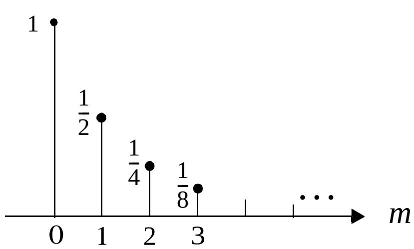
(+2 pts.) $\boxed{A = 0,}$ $b = 1, c = 2,$ $\boxed{d = 3}$ (+2 pts.)

$$s(n+1) = x(n)$$
$$y(n) = 2s(n) + 3x(n)$$

or $A = 0, b = 2, c = 1, d = 3$

$\boxed{b \cdot c = 2}$ (+2 pts)

[6] (c)

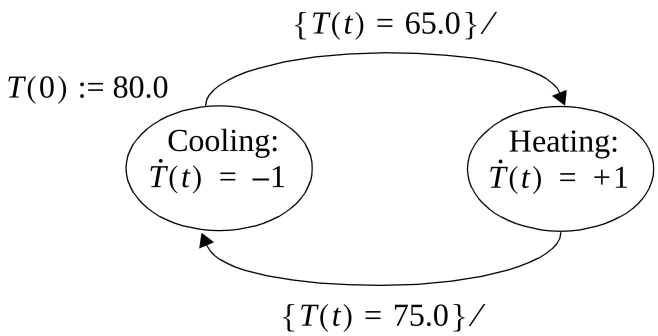


$$s(n+1) = \frac{1}{2}s(n) + 1x(n) \qquad \text{(3 pts.)}$$
$$y(n) = \frac{1}{2}s(n) + 1x(n) \qquad \text{(3 pts.)}$$

[18 pts.] Question 3

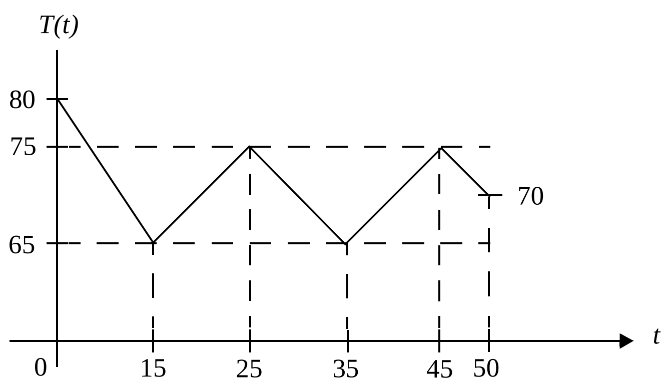
This problem is to design a hybrid system that models how a thermostat can keep the room temperature of a house between 65° and 75°.

As shown in the figure below, this hybrid system has two operation modes, Cooling and Heating, corresponding to whether to cool or heat the room. The state variable $T(t)$ indicates the room temperature. In each mode there is a differential equation that describes how the temperature changes with respect to time. The thermostat is turned on at time 0.0 and $T(0)$ is 80.0:



[5] a) Supposing that the initial mode of the thermostat is Cooling, plot the signal $T(t)$ with t from 0 to 50.

(+5 pts.)

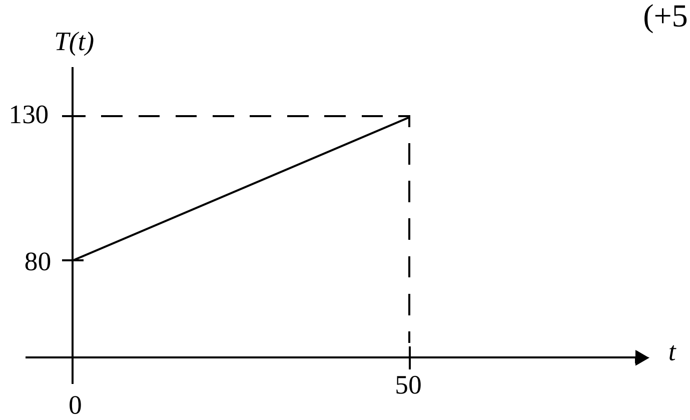


Note:

- no values on axis (–2 pts.)
- discrete not continuous-time (–2 pts.)

[5] b) Supposing the initial mode of the thermostat is Heating, plot the signal $T(t)$ with t from 0 to 50.

(+5 pts.)



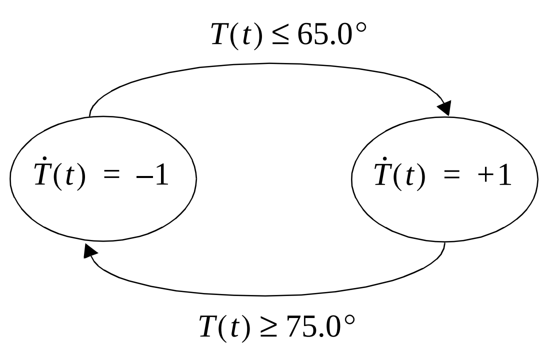
Note:

- no values (–2 pts.)

[8]c) Supposing $T(0)$ is any real number, can this hybrid-system model of a thermostat always make the room temperature eventually fall in the range [65, 75]? If yes, explain why. If not, modify the model so that it always meets this goal.

No (+2 pts.)

(+ 6 pts.)



[24 pts.] Question 4

[7] a) A system with input x and output y is given by a 1st order differential equation:

$$\dot{y}(t) + ay(t) = bx(t), \qquad t \geq 0.$$

What are the initial conditions that need to be specified? Give a discrete-time approximation of the system that can be simulated on a computer as a linear state machine.

Initial condition $y(0)$ (+2 pts.)

$$s(n+1) = (1 - a\Delta)s(n) + b\Delta x_d(n)$$
$$y_d(n) = s(n), \qquad s(0) = y(0)$$
(+3 pts.)

where $x_d(n) = x(n\Delta)$ and $y_d(n) = y(n\Delta)$ (+2 pts.)

[7] b. A system with input x and output y is given by:

$$y(t) = \int_0^t x(s)ds, \qquad t \geq 0.$$

What are the initial conditions that need to be specified? Give a discrete-time approximation of the system that can be simulated on a computer as a linear state machine..

No initial condition needs to be specified.

$$y(0) = \int_0^0 x(s)ds = 0$$ (+2 pts.)

$$s(n+1) = s(n) + \Delta x_d(n)$$
$$y_d(n) = s(n), \qquad s(0) = 0$$
(+3 pts.)

where $x_d(n) = y(n\Delta)$ and $x_d(n) = x(n\Delta)$ (+2 pts.)

[10] c) A system with input x and output y is given by:

$$\dot{y}(t) + ay(t) = \int_0^t x(s)ds, \qquad t \geq 0 .$$

What are the initial conditions that need to be specified? Give a discrete-time approximation of the system that can be simulated on a computer as a linear state machine.

- Initial condition: We have $\dot{y}(0) + ay(0) = \int_0^0 x(s)ds = 0$ (+2 pts.)

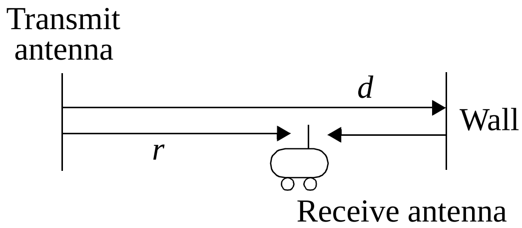
  so we need either $\dot{y}(0)$ or $y(0)$, but not both. (+2 pts.)

- $s(n) = \begin{bmatrix} \dot{y}_d(n) \\ y_d(n) \end{bmatrix}$ where $\begin{matrix} y_d(n) = y(n\Delta) \\ \dot{y}_d(n) = \dot{y}(n\Delta) \end{matrix}$ (+3 pts.)

  $s(n+1) = \begin{bmatrix} 1 - a\Delta & 0 \\ \Delta & 1 \end{bmatrix} s(n) + \begin{bmatrix} \Delta \\ 0 \end{bmatrix} x(n)$ (+3 pts.)

[38 pts.] Question 5

A wireless communication scenario is given in the figure below:



The receiver is on a stationary vehicle. The signal is received both along the directed path and the reflected path. The signal received along each path is delayed by the time it takes for light to travel along that path. The reflection causes a sign change in the signal as well. You can assume for simplicity that there is no attenuation of the signal along either path. The received signal is the superposition of the signals along each path.

[6] a) Give the relationship between the input x at the transmitter and the output y at the receiver. You can express your answer in terms of c, the speed of light. Is the system linear? Time invariant? Explain.

$$y(t) = x(t-\tau_1) - x(t-\tau_2) \quad \text{where} \quad \tau_1 = \frac{r}{c} \text{ and } \tau_2 = \frac{2d-r}{c} \qquad \text{(+2 pts.)}$$

Linear (+2 pts.)

TI (+2 pts.)

[6] b) Suppose $x(t) = \cos(2\pi f t)$. Verify that the output is also a sinusoid at the same frequency. You can use the trigonometric identity:

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right).$$

$$\tau_1 - \tau_2 = \frac{2r-2d}{c} \qquad\qquad \tau_1 - \tau_2 = \frac{2d}{c}$$

$$y(t) = -2\sin[\pi f(\tau_1 - \tau_2)]\sin\left[2\pi f\left(t - \frac{d}{c}\right)\right]$$

$$= 2\sin\left[2\pi f\frac{(d-r)}{c}\right]\sin\left[2\pi f\left(t-\frac{d}{c}\right)\right] \qquad \text{(+3 pts.)}$$

$\sin\left[2\pi f\frac{(d-r)}{c}\right]$ is constant

(+3 pts.)

$\sin\left[2\pi f\left(t-\frac{d}{c}\right)\right]$ has the same frequency as f

[6] c) We can use the amplitude of the output as a measure of the received signal strength. What are the locations at which the signal strength is the maximum? What are the locations at which the signal strength is minimum? How far do you have to go from a maximum point to a minimum point? How does that depend on the frequency f?

$$\text{amplitude} = \left|2\sin\left[2\pi f\frac{(d-r)}{c}\right]\right|$$

when $2\pi f\dfrac{d-r}{c} = \dfrac{2k+1}{2}\pi \qquad k \in \text{Integer}$

amplitude has maximum value

$$\Rightarrow r_{max} = d - \frac{c}{4f}(2k+1) \qquad \text{(+2 pts.)}$$

when $2\pi f\dfrac{d-r}{c} = k\pi \qquad k \in \text{Integer}$

amplitude has minimum value

$$\Rightarrow r_{min} = d - \frac{c}{2f}k \qquad \text{(+2 pts.)}$$

The minimum distance from a maximum point to a minimun point is

$$\left|r_{max} - r_{min}\right|_{k=0} = \frac{c}{4f} \qquad \text{(+1 pt.)}$$

This distance is $\alpha\dfrac{1}{f}$ (+1 pt.)

[6] d) Suppose now that the vehicle is moving towards the wall at speed v such that the position at time t is $r(t) = r_0 + vt$. Give the input/output relationship. Is the system linear? Time invariant? Explain.

$$\tau_1(t) = \frac{r_0 + vt}{c} \qquad \tau_2(t) = \frac{2d - r_0 - vt}{c}$$

$$y(t) = x(t - \tau_1(t)) - x(t - \tau_2(t)) \qquad \text{(+2 pts.)}$$

Linear (+2 pts.)

Not TI (+2 pts.)

[6] e) What is now the output given an input $x(t) = \cos 2\pi f t$? What is the frequency content of the received signal?

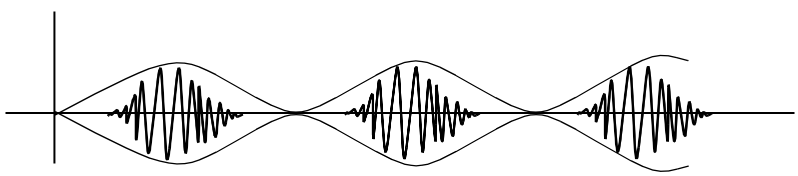
$$y(t) = \cos\left[2\pi f\left(t - \frac{v}{c}t - \frac{r_0}{c}\right)\right] - \cos\left[2\pi f\left(t + \frac{vt}{c} + \frac{r_0}{c} - \frac{2d}{c}\right)\right] \qquad \text{(+3 pts.)}$$

Two frequencies: $\left(1 - \frac{v}{c}\right)f$ and $\left(1 + \frac{v}{c}\right)f$ (+3 pts.)

[6] f) Express the output as a product of two sinusoids and give a sketch of the output. You can assume that $v \ll c$.

$$y(t) = -2\sin\left(2\pi f\,\frac{r_0 + vt - d}{c}\right)\cos\left[2\pi f\left(t - \frac{d}{c}\right)\right] \qquad \text{(+3 pts.)}$$

This is a product of two sinusoids, one at frequency f and one at frequency fv/c. The latter is at a much lower frequency and appears as an oscillating envelope in the plot below:



(+3 pts.)

[2] g) Can you reconcile your sketch in (f) with the phenomenon described in part (c)?

Envelope varies as the vehicle moves from max to min amplitude points. (+2 pts.)