EECS 20. Final Exam Solution
May 15, 2000.

1. (a) Linear: all except S_3.
   (b) Time invariant: S_1, S_2, and S_3.
   (c) Causal: S_1, S_3 and S_6.

2. (a) Since the system is causal, $h(n) = 0$ for $n < 0$. In addition, h satisfies

$$h(n) = \delta(n) + \delta(n-1) - \alpha h(n-1)$$

(just let the input be an impulse). Thus,

$$\begin{aligned} h(0) &= 1 \\ h(1) &= (1-\alpha) \\ h(2) &= -\alpha(1-\alpha) \\ h(3) &= \alpha^2(1-\alpha) \\ h(4) &= -\alpha^3(1-\alpha) \\ &\cdots \\ h(n) &= (-\alpha)^{n-1}(1-\alpha) \end{aligned}$$

so

$$h(n) = (-\alpha)^{n-1}u(n-1) + (-\alpha)^n u(n),$$

where $u(n)$ is the unit step function.

   (b) Although we could calculate the DTFT of the impulse response, it is easier to just let the input be a complex exponential,

$$x(n) = e^{i\omega n}.$$

The output then will be

$$y(n) = H(\omega)e^{i\omega n}.$$

Hence, the following equation must be satisfied,

$$H(\omega)e^{i\omega n} + \alpha H(\omega)e^{i\omega(n-1)} = e^{i\omega n} + e^{i\omega(n-1)}.$$

We can factor out $e^{i\omega n}$ and divide through by it, getting

$$H(\omega)(1 + \alpha e^{-i\omega}) = 1 + e^{-i\omega}.$$

Hence,

$$H(\omega) = \frac{1 + e^{-i\omega}}{1 + \alpha e^{-i\omega}}.$$

   (c) The output will be zero if the frequency ω of the sinusoid is such that $H(\omega) = 0$. This occurs if $e^{-i\omega} = -1$, which occurs if $\omega = \pi$. Thus, the following input will yield zero output:

$$x(n) = \cos(\pi n).$$

(d)
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omega = [0: pi/400: pi];
H = (1 + exp(-i*omega))./(1 + alpha*exp(-i * omega));
plot(omega, abs(H));
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(e) A reasonable choice for the state s is

$$s(n) = [x(n-1), y(n-1)]^T.$$

With this choice,

$$A = \begin{bmatrix} 0 & 0 \\ 1 & -\alpha \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 \\ -\alpha \end{bmatrix}, \quad d = 1.$$

(f) If $\alpha = 1$, the frequency response becomes $H(\omega) = 1$ and the impulse response becomes $h(n) = \delta(n)$.

3. (a) False. The output frequency may not be the same as the input frequency.

   (b) False. You only know the response to one frequency.

   (c) True. The frequency response is the DTFT of the impulse response.

   (d) True. The impulse response is $y(n) - y(n-1)$, from which you can determine the frequency response.

   (e) True. If the system were LTI, the response to the delayed impulse would be the delayed impulse response.

   (f) False. The system might be LTI with impulse response given by $h(n) = y(n) - y(n-2) + y(n-4) - y(n-6) + \cdots$.

4. (a) The fundamental frequency is

$$\omega_0 = 10\pi \text{ radians/second}.$$

   (b) The Fourier series coefficients are

$$A_0 = 0, A_1 = 1, A_2 = 1, A_3 = 1, A_k = 0 \text{ for } k > 3,$$

and

$$\phi_k = 0 \text{ for all } k.$$

   (c) The sampled signal is

$$\begin{aligned} y(n) &= \cos(10\pi n/10) + \cos(20\pi n/10) + \cos(30\pi n/10) \\ &= 1 + 2\cos(\pi n). \end{aligned}$$

The fundamental frequency is therefore $\omega_0 = \pi$ radians/sample.

   (d) The DFS coefficients are

$$A_0 = 1, A_1 = 2, \phi_1 = 0.$$

There are no more coefficients, since the period is $p = 2$.

   (e) The "smoothest" (lowest frequency content) interpolating signal is

$$w(t) = 1 + 2\cos(10\pi t).$$

(f) Yes, there is aliasing distortion. The 10 Hz cosine has been aliased down to DC, and the 15 Hz cosine has been aliased down to 5 Hz, overlapping the 5 Hz cosine.

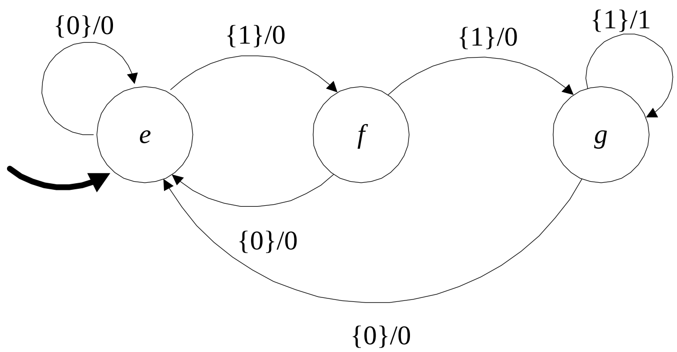
(g) Sampling at twice the highest frequency will work. The highest frequency is 15 Hz, so sampling at 30 Hz will avoid aliasing distortion.

5. The sawtooth signal has period $p = 1$ second, so its fundamental frequency is 2π radians/second, considerably above the passband of the filter. Thus, only the DC term gets through the filter. The DC term is the average over one period, which is 1/2, so the output is

$$y(n) = 1/2.$$

6. (a) False.
   (b) True.
   (c) True.
   (d) True.
   (e) False.

7. The machine is shown below:



The simulation relation is

$$\{(a, e), (b, f), (c, g), (d, g)\}.$$