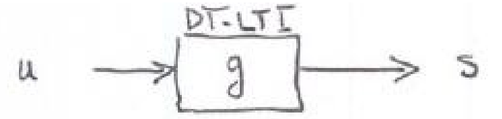
EE 20N, Fall, 2005, Midterm 2, Ayazifar

MT2.1

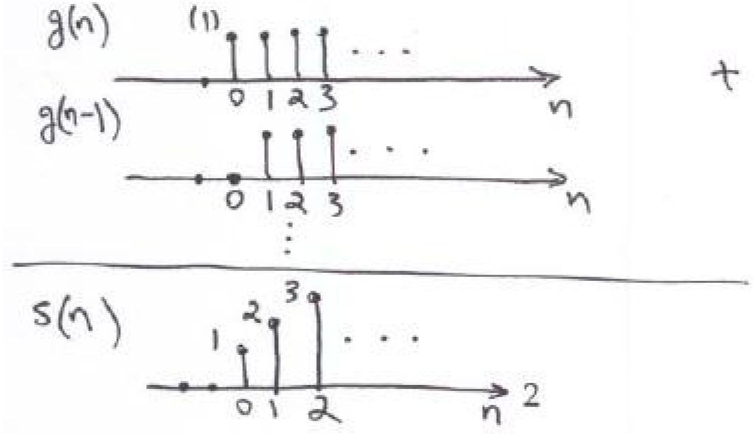


$\underline{n \geq 0}$

$$s(n) = (u * g)(n) = \sum_{k=-\infty}^{\infty} u(k)\, g(n-k) = \sum_{\underset{\text{b/c } u(k)=0\ \forall k<0}{k=0}}^{\overset{\text{b/c } g(n-k)=0\ \forall k>n}{n}} 1 = n+1$$

$$s(n) = \begin{cases} n+1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

<u>Graphically</u>: Using the "echo" view of convolution,
s(n) = g(n) + g(n-1) + g(n-2) + … = u(n) + u(n-1) + u(n-2) + …

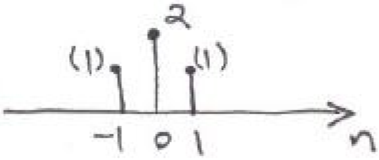


MT2.2.a
No.
$x_1(n) = x_2(n+1)$ for all $n \leq 1$, but $y_2(n+1) \neq 0$ for $n = -1$ (whereas $y_1(n) = 0$ at $n = -1$)

MT2.2.b
No. A system that is not causal cannot be memoryless.
Recall that: memoryless → causal, which is logically
Equivalent to: not causal → not memoryless

MT2.2.c

F is time invariant. Hence, knowing that $\delta(n) = x_3(n+4)$ means that the response of the system to the unit impulse is $(F(\delta))(n) = y_3(n+4)$.



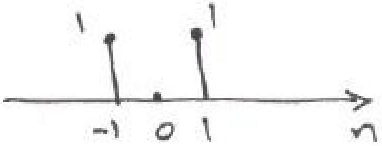
MT2.2.d

Not linear.

$\delta(n) = x_1(n) - x_2(n+1)$

F is TI → F's response to w(n), s.t. $w(n) = x_2(n+1)$ is $\hat{y}_2$, s.t. $\hat{y}_2(n) = y_2(n+1)$.

Assume F is linear. Then $y_1(n) - y_2(n+1)$ is plotted below:



This is not the same as $(F(\delta))(n)$ found in part (c). This is a contradiction. Therefore, F cannot be linear.

MT2.3.a

$$s(1) = A\,s(0) = \alpha \begin{bmatrix} 1/2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1/2 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \alpha \left(\frac{1}{2}\right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta (2) \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\vdots$$

$$s(n) = \alpha \left(\frac{1}{2}\right)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta (2)^n \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

MT2.3.b

$$s(n) = 2 \left(\frac{1}{2}\right)^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2^n \begin{bmatrix} 2 \\ 3 \end{bmatrix} \Rightarrow s(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

The second component of s(n) will be larger than the second component of s(1) (which is 6). Therefore, $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$ is not reachable

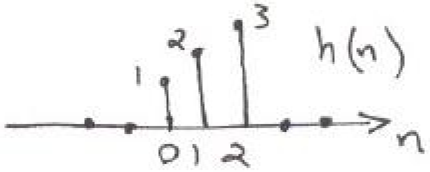
MT2.3.c

s(n) has two terms: a “stable” term $\alpha\left(\frac{1}{2}\right)^n\begin{bmatrix}1\\0\end{bmatrix}$, which decays to zero as n → ∞;

and an “unstable” term $\beta(2)^n\begin{bmatrix}2\\3\end{bmatrix}$, which grows exponentially as n → ∞. Hence, we must have: $\alpha \epsilon R$ $\beta = 0$

MT2.4.a
$y(n) = x(n) + 2x(n-1) + 3x(n-2)$

MT2.4.b
$h(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-2)$



MT2.4.c

$$\begin{aligned} s_1(n+1) &= s_2(n) \\ s_2(n+1) &= x(n) \end{aligned} \Rightarrow \begin{bmatrix} s_1(n+1)\\ s_2(n+1)\end{bmatrix} = \underbrace{\begin{bmatrix}0&1\\0&0\end{bmatrix}}_{A}\begin{bmatrix}s_1(n)\\s_2(n)\end{bmatrix} + \underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{B} x(n)$$

$$y(n) = x(n) + 2s_2(n) + 3s_1(n) \Rightarrow y(n) = \underbrace{\begin{bmatrix}3 & 2\end{bmatrix}}_{C}\begin{bmatrix}s_1(n)\\s_2(n)\end{bmatrix} + \underset{\overset{\uparrow}{D}}{1}\cdot x(n)$$

MT2.5

$$p=4 \implies \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

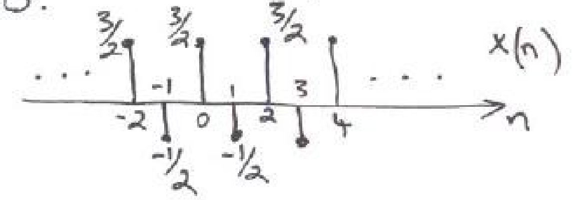
$$X_0 = \frac{1}{4}\sum_{n=-1}^{2} x(n) = \frac{1}{4}(2) = \frac{1}{2}$$

$$X_2 = \frac{1}{4}\sum_{n=-1}^{2} x(n) e^{-i\cancel{2}\frac{\pi}{\cancel{2}}n} = \frac{1}{4}\sum_{n=-1}^{2} (-1)^n x(n) = \frac{1}{4}(4) = 1$$

$$X_1 = \frac{1}{4}\sum_{n=-1}^{2} x(n) e^{-i\frac{\pi}{2}n} = \frac{1}{4}\left(\cancelto{0}{\sum_{n=-1}^{2} x(n)\cos\left(\frac{\pi}{2}n\right)} - i\cancelto{0}{\sum_{n=-1}^{2} x(n)\sin\left(\frac{\pi}{2}n\right)}\right) = 0$$

$X_{-1} = 0$ for same reason as $X_1 = 0$.

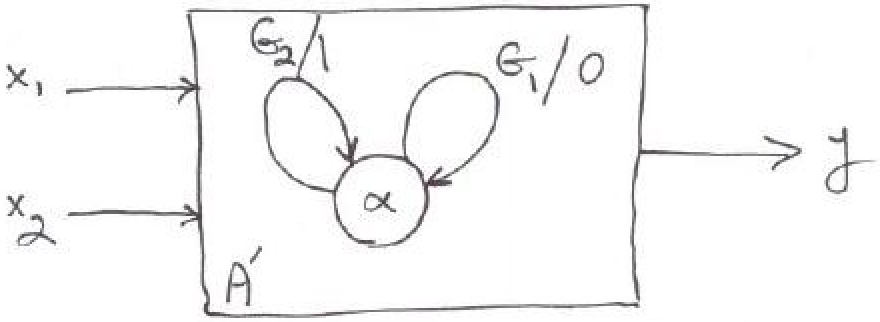
Hence, $x(n) = \frac{1}{2} + (-1)^n$ 7



MT2.6.a.i

Yes. It doesn’t matter what state you’re in. This machine is bisimilar to a one-state machine (see part (ii)).

MT2.6.a.ii



The answer to this part should have confirmed your answer to part (i).

MT2.7.b.i

(I): B is not well-formed because for the same $x_1 = 1$, there are two non-stuttering fixed points.

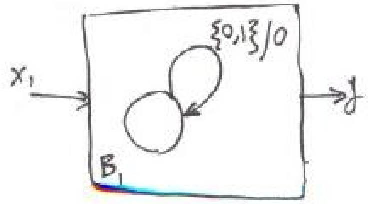
(II),(III): B is well-formed because for each non-stuttering input x_1, there is a unique fixed point:

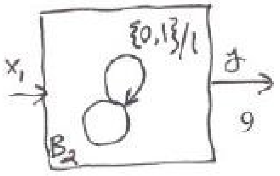
II) $x_1 = 0 \rightarrow y = x_2 = 1$
$x_1 = 1 \rightarrow y = x_2 = 1$
III) $x_1 = 0 \rightarrow y = x_2 = 0$
$x_1 = 1 \rightarrow y = x_2 = 0$

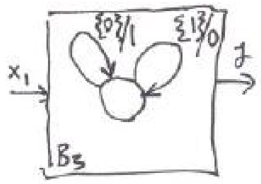
(IV): B is not well-formed. No non-stuttering fixed point for $x_1 = 0$.

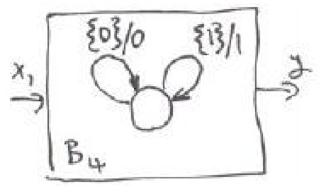
MT2.7.b.ii

If B is well-formed, it is bisimilar to one of the following deterministic single-state machines:









Each of these is memoryless.