EECS20n, Midterm 1, 10/20/00

Please print your name and your TA's name here:

Last Name _____ First _____ TA's name _____

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Problem 6:

Problem 7:

Problem 8:

Total:

Read the questions carefully before you answer. Good luck.

1. **10 points** The function x : *Reals* $\rightarrow$ *Reals* given by

$$\forall t \in \textit{Reals} \quad x(t) = \sin(2\pi \times 440t)$$

is a mathematical example of a signal in the signal space [*Reals* $\rightarrow$ *Reals*].

Give a mathematical example of a signal x in each of the following signal spaces.

(a) [*Ints* $\rightarrow$ *Reals*]

(b) [$\textit{Nats}_0 \rightarrow$ *EnglishWords*]

(c) [*Reals* $\rightarrow \textit{Reals}^2$]

(d) $[\{0, 1, \cdots, 600\} \times \{0, 1, \cdots, 400\} \rightarrow \{0, 1, \cdots, 255\}]$

(e) Give an example of a practical space of signals whose mathematical representation is $[\{0, 1, \cdots, 600\} \times \{0, 1, \cdots, 400\} \rightarrow \{0, 1, \cdots, 255\}]$.

2. **10 points** The function H : [*Reals*$_+$ $\rightarrow$ *Reals*] $\rightarrow$ [*Nats*$_0$ $\rightarrow$ *Reals*] given by: $\forall x \in$ [*Reals*$_+$ $\rightarrow$ *Reals*],

$$\forall n \in \textit{Nats}_0, \quad H(x)(n) = x(10n),$$

is a mathematical example of a system with input signal space [*Reals*$_+$ $\rightarrow$ *Reals*] and output signal space [*Nats*$_0$ $\rightarrow$ *Reals*]. Give a mathematical example of a system H whose

(a) input and output signal spaces both are [*Nats*$_0$ $\rightarrow$ *Bin*].

(b) input signal space is [*Nats*$_0$ $\rightarrow$ *Reals*] and output signal space is [*Nats*$_0$ $\rightarrow \{0, 1\}$].

(c) input signal space is [*Ints* $\rightarrow$ *Reals*] and output signal space is [*Reals* $\rightarrow$ *Reals*].

3. **10 points** A state machine has *Inputs* $=$ *Outputs* $= \{0, 1\}$.

   (a) What is the space of input signals and the space of output signals of this state machine?

   (b) Construct a *deterministic* machine whose input-output function H is given by (letting x denote the input signal and $y = H(x)$ denote the output signal): $\forall n \geq 0$,

$$y(n) = \begin{cases} 0, & \text{if } n = 0, 1 \\ x(n-2), & \text{if } n \geq 2 \end{cases}$$

   (c) What is the output of your machine when the input is $0, 1, 0, 1, \cdots$?

4. **10 points** Construct a *non-deterministic* state machine with *Inputs* $=$ *Outputs* $= \{T, F\}$ which for any input signal x has two possible output signals y, namely $y = x$, and $y = \bar{x}$ where $\forall n, \quad \bar{x}(n) = T$ or F accordingly as $x(n) = F$ or T.

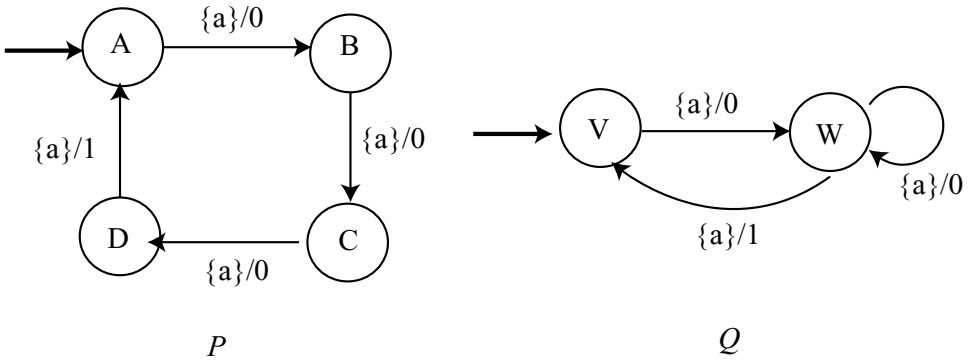

Figure 1: Q simulates P

5. **10 points** Let

$$M = (\mathit{States}_M, \mathit{Inputs}, \mathit{Outputs}, \mathit{possibleUpdates}_M, \mathit{initialState}_M),$$

$$N = (\mathit{States}_N, \mathit{Inputs}, \mathit{Outputs}, \mathit{possibleUpdates}_N, \mathit{initialState}_N),$$

be two non-deterministic state machines with the same set of inputs and outputs. Let $S \subset \mathit{States}_M \times \mathit{States}_N$.

(a) Give the definition for S to be a simulation relation.

(b) Find the simulation relation between P and Q shown in figure 1. Here $\mathit{Inputs} = \{a\}$ and $\mathit{Outputs} = \{0, 1\}$. (In the figure P is deterministic.)

(c) Are P and Q in figure 1 bisimilar? Answer yes or no.

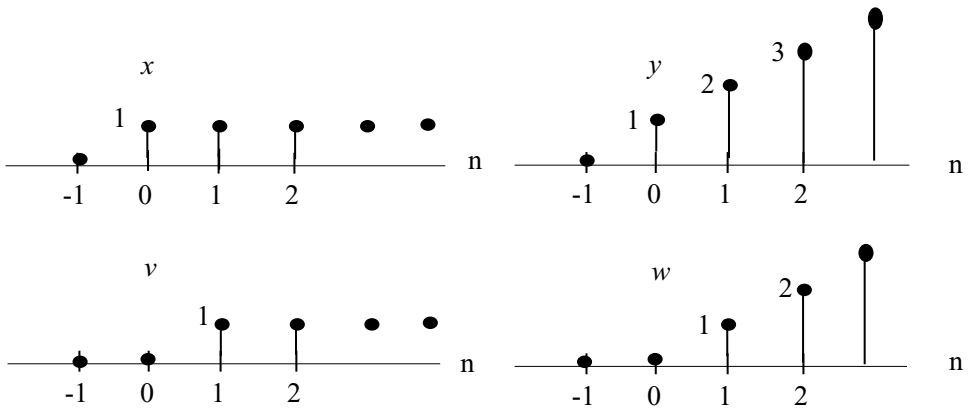

Figure 2: Results of two experiments

6. **10 points** Consider a multidimensional SISO system

$$\begin{aligned} s(n+1) &= As(n) + bx(n) \\ y(n) &= c^T s(n) + dx(n) \end{aligned}$$

Suppose you don't know A, b, c, d or the initial state $s(0)$. Two input-output experiments are performed. In the first experiment, the input signal is x and the output signal is y; in the second, the input signal is v and the output signal is w. These signals are shown in figure2. Mathematically, they are:

$$x(n) = 1, n \geq 0, = 0, n < 0; \;\; y(n) = n + 1, n \geq 0, = 0, n < 0;$$
$$v(n) = 1, n \geq 1, = 0, n < 1; \;\; w(n) = n, n \geq 0, = 0, n < 0.$$

In both cases the initial state $s(0)$ is the same.

(a) What is the zero-state impulse response of the system?

(b) What is the zero-state step response, i.e. the zero-state response of the system to the input signal x?

(c) What is the zero-input response, i.e. the response when the input signal is identically zero ($s(0)$ is still the initial state).

7. **10 points** Answer the following True/False questions about a system

$$H : [\textit{Ints} \to \textit{Reals}] \to [\textit{Ints} \to \textit{Reals}]$$

In each case a correct answer yields +2 points, an incorrect answer yields -2 points, no answer yields 0 points.

(a) If

$$\forall x, \forall n, \quad (H(x))(n) = x(-n), \tag{1}$$

H is linear.

(b) The system (1) is time-invariant.

(c) If

$$\forall x, \forall n, \quad (H(x))(n) = x^2(n) - x^2(n-1), \tag{2}$$

H is linear.

(d) The system (2) is time-invariant.

(e) The system given by

$$\forall x, \forall n, \quad (H(x))(n) = 0.5x(n) + 0.2x(n-3),$$

is linear and time-invariant.

8. **10 points** Construct a linear time-invariant system of the form,

$$\begin{aligned} s(n+1) &= As(n) + bx(n) \\ y(n) &= c^T s(n) + dx(n), \end{aligned}$$

whose zero-state impulse response h is given by: $h(0) = 3, h(1) = -2$, and $h(n) = 0$, otherwise.