EE 16B Final, December 13, 2016

Name: SOLUTIONS

SID #: __________________

Important Instructions:

- **Show your work.** An answer without explanation is not acceptable and does not guarantee any credit.
- **Only the front pages will be scanned and graded.** If you need more space, please ask for extra paper instead of using the back pages.
- **Do not remove pages**, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

Problem	**Points**	**Score**
1	15	
2	15	
3	15	
4	15	
5	10	
6	10	
7	10	
8	10	
Total	100	

1. (15 points) Consider the discrete-time system

$$\vec{x}(t+1) = A\vec{x}(t) + Bu(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

a) (3 points) Show that the system is controllable.

$$AB = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[A^2B \,\vdots\, AB \,\vdots\, B\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \text{full rank, therefore controllable}$$

b) (5 points) We wish to move the state vector from $\vec{x}(0) = 0$ to

$$\vec{x}(T) = \begin{bmatrix}2\\0\\0\end{bmatrix}.$$

Find the smallest possible time T and an input sequence $u(0), \ldots, u(T-1)$ to accomplish this task.

$$\underbrace{\vec{X}(T)}_{\begin{bmatrix}2\\0\\0\end{bmatrix}} = A^T \underbrace{\vec{X}(0)}_{=0} + \underbrace{[A^{T-1}B \ \cdots \ AB \ \ B]}_{\substack{= I \\ \text{for } T=3 \\ \text{from part (a)}}} \begin{bmatrix}U(0)\\ \vdots \\ U(T-1)\end{bmatrix}$$

Choose $\boxed{\begin{bmatrix}U(0)\\U(1)\\U(2)\end{bmatrix} = \begin{bmatrix}2\\0\\0\end{bmatrix}.}$ Then $\vec{X}(3) = \begin{bmatrix}2\\0\\0\end{bmatrix}$ as desired.

$T=2$ won't work because

$$\vec{X}(2) = \underbrace{[AB \ \ B]}_{\begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}} \begin{bmatrix}U(0)\\U(1)\end{bmatrix} = \begin{bmatrix}0\\U(0)\\U(1)\end{bmatrix} \neq \begin{bmatrix}2\\0\\0\end{bmatrix}$$

for any choice of $u(0), u(1)$.

Therefore, $\boxed{T=3 \text{ is the smallest possible.}}$

c) (7 points) Now suppose that the control input is subject to the constraint $|u(t)| \leq 1$ for all t; that is, we can't apply inputs with magnitude over 1. Find the smallest possible T under this constraint and an input sequence $u(0), \ldots, u(T-1)$ such that $\vec{x}(T)$ is as specified in part (b).

In part (b), we found $u(0)=2$ which violates the constraint $|u| \leq 1$.

Note that $A^3B = B$, $A^4B = AB$, $A^5B = A^2B$.

Therefore,

$$\vec{x}(6) = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccccc} A^5B & A^4B & A^3B & A^2B & AB & B \end{array} \end{array} \begin{bmatrix} u(0) \\ u(1) \\ u(2) \\ u(3) \\ u(4) \\ u(5) \end{bmatrix} = \begin{bmatrix} u(0)+u(3) \\ u(1)+u(4) \\ u(2)+u(5) \end{bmatrix}$$

Choose $\boxed{\begin{aligned} u(0)&=u(3)=1 \\ u(1)&=u(4)=0 \\ u(2)&=u(5)=0 \end{aligned}}$

so that $\vec{x}(6) = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $|u| \leq 1$ is satisfied.

$\boxed{T=6 \text{ is the smallest possible:}}$

$$\vec{x}(5) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u(0) \\ u(1) \\ u(2) \\ u(3) \\ u(4) \end{bmatrix} = \begin{bmatrix} u(2) \\ u(0)+u(3) \\ u(1)+u(4) \end{bmatrix}$$

To match to $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ we need $u(2)=2$ which violates $|u| \leq 1$.

2. (15 points) In parts (a) and (b) below find a singular value decomposition,

$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T.$$

a) (5 points)
$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$\sigma_1 = 1$ $\quad \vec{u}_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ $\quad \vec{v}_1^T = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$\sigma_2 = 1$ $\quad \vec{u}_2 = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$ $\quad \vec{v}_2^T = \begin{bmatrix} 0 & 1 \end{bmatrix}$

ALSO CORRECT if $\vec{V}_1, \vec{V}_2$ any orthonormal pair and $\vec{U}_1, \vec{U}_2$ satisfy $\vec{U}_1 = A\vec{V}_1$, $\vec{U}_2 = A\vec{V}_2$.

$$A^TA = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\lambda_1 = \lambda_2 = 1$ and any orthonormal $\vec{V}_1, \vec{V}_2$ will serve as evectors, say $\vec{V}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\vec{V}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\sigma_1 = \sqrt{\lambda_1} = 1$ $\quad \vec{U}_1 = \frac{1}{\sigma_1} A\vec{V}_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ $\quad \vec{U}_2 = \frac{1}{\sigma_2} A\vec{V}_2 = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$

$\sigma_2 = \sqrt{\lambda_2} = 1$

b) (6 points)

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$\sigma_1 = 3$ $\vec{u}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ $\vec{v}_1^T = \frac{1}{3\sqrt{2}}\begin{bmatrix}1 & 2 & 2 & 2 & 2 & 1\end{bmatrix}$

$\sigma_2 = 1$ $\vec{u}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ $\vec{v}_2^T = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0 & 0 & 0 & 0 & -1\end{bmatrix}$

Also correct if both $\vec{u}_1$ and $\vec{v}_1$ multiplied by (-1). Same for $\vec{u}_2$ and $\vec{v}_2$.

$$AA^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Evalues of AA^T:

$$\begin{vmatrix} \lambda-5 & -4 \\ -4 & \lambda-5 \end{vmatrix} = (\lambda-5)^2 - 4^2 = 0 \Rightarrow \lambda = 5 \mp 4$$

$$\underline{\lambda_1 = 9}, \quad \underline{\lambda_2 = 1}$$

Evectors:

$$(\lambda_1 I - A)\vec{u}_1 = 0 \qquad (\lambda_2 I - A)\vec{u}_2 = 0$$

$$\begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix}\vec{u}_1 = 0 \qquad \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}\vec{u}_2 = 0$$

Choose $\vec{u}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ $\vec{u}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

Not unique: can choose negative signs

to normalize

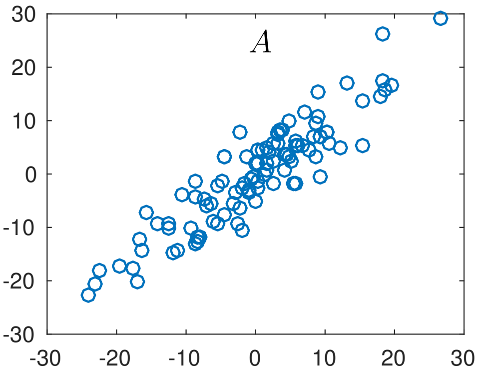
$$\sigma_1 = \sqrt{\lambda_1} = 3$$

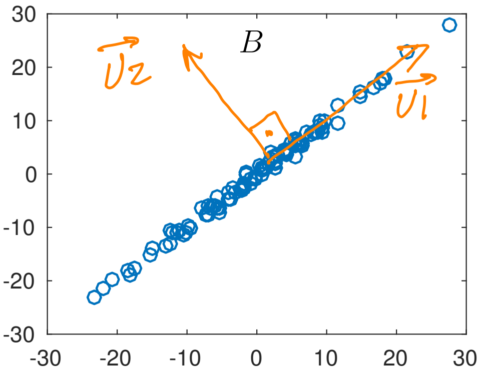
$$\vec{v}_1 = \frac{1}{\sigma_1}A^T\vec{u}_1 = \frac{1}{3\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{3\sqrt{2}}\begin{bmatrix}1\\2\\2\\2\\2\\1\end{bmatrix}$$

$$\sigma_2 = \sqrt{\lambda_2} = 1$$

$$\vec{v}_2 = \frac{1}{\sigma_2}A^T\vec{u}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\\0\\-1\end{bmatrix}$$

c) (4 points) The plots below correspond to two 2×100 matrices, A and B. Each of the 100 points on the left plot represents a column vector of A and the right plot is constructed similarly for B. Answer the questions below based on the qualitative features of the plots rather than precise numerical values.





i) (2 points) Which matrix has the **biggest** ratio σ_1/σ_2 of the largest singular value σ_1 to the second singular value σ_2?

Answer: B (less spread around line – see Lecture 9B)

ii) (2 points) Go to the plot A or B corresponding to your answer in part (i), and draw one line that shows the direction of the vector $\vec{u}_1$ and another line that shows the direction of the vector $\vec{u}_2$ in the singular value decomposition

$$A = \sigma_1 \vec{u}_1 \vec{v}_1^T + \sigma_2 \vec{u}_2 \vec{v}_2^T.$$

Clearly label each line to indicate which one is for $\vec{u}_1$ and which one for $\vec{u}_2$.

3. (15 points) The continuous-time functions in parts (a)-(e) below are sampled with period $T = 1$. For each one determine whether the function can be recovered from its samples by sinc interpolation. If your answer is no, indicate what other continuous function would result from sinc interpolation.

a) $\cos\left(\frac{3\pi}{4}t\right)$ Yes.

$$\omega = \frac{3\pi}{4} < \frac{\pi}{T} = \pi$$

therefore sampling theorem (Lectures 11A-B) guarantees recovery from samples by sinc interpolation.

b) $\cos\left(\frac{5\pi}{4}t\right)$ No.

$$\omega = \frac{5\pi}{4} > \pi.$$

Note $\cos(2\pi t - \theta) = \cos\theta$ for all integers t

therefore

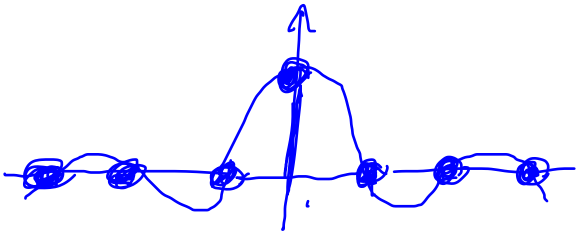
$$\cos\Big(\underbrace{2\pi t - \frac{5\pi}{4}t}_{\frac{3\pi}{4}t}\Big) = \cos\left(\frac{5\pi}{4}t\right).$$

Thus the functions in parts a and b give the same exact samples. We know from part (a) that

sinc interpolation from these samples gives $\cos\left(\frac{3\pi}{4}t\right)$.

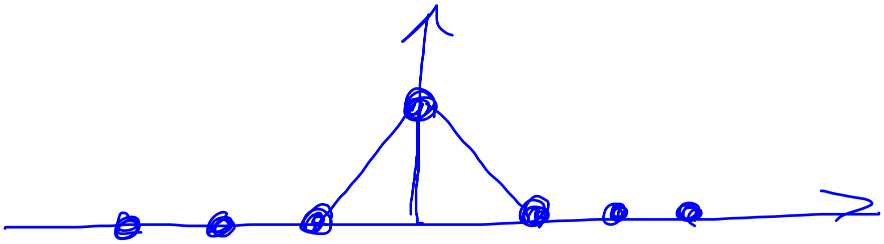
For parts (c)-(e) below sketching the samples of the function with period $T = 1$ may help in determining whether the function can be recovered from these samples and, if not, what function results from sinc interpolation.

c) $\text{sinc}(t) \triangleq \begin{cases} \frac{\sin(\pi t)}{\pi t} & t \neq 0 \\ 1 & t = 0 \end{cases}$ Yes.



All samples are zero, except at t=0.
Therefore, sinc interpolation gives
sinc(t)
which is the sampled function.

d) $f(t) = \begin{cases} 1 - |t| & |t| \leq 1 \\ 0 & \text{otherwise.} \end{cases}$ No.



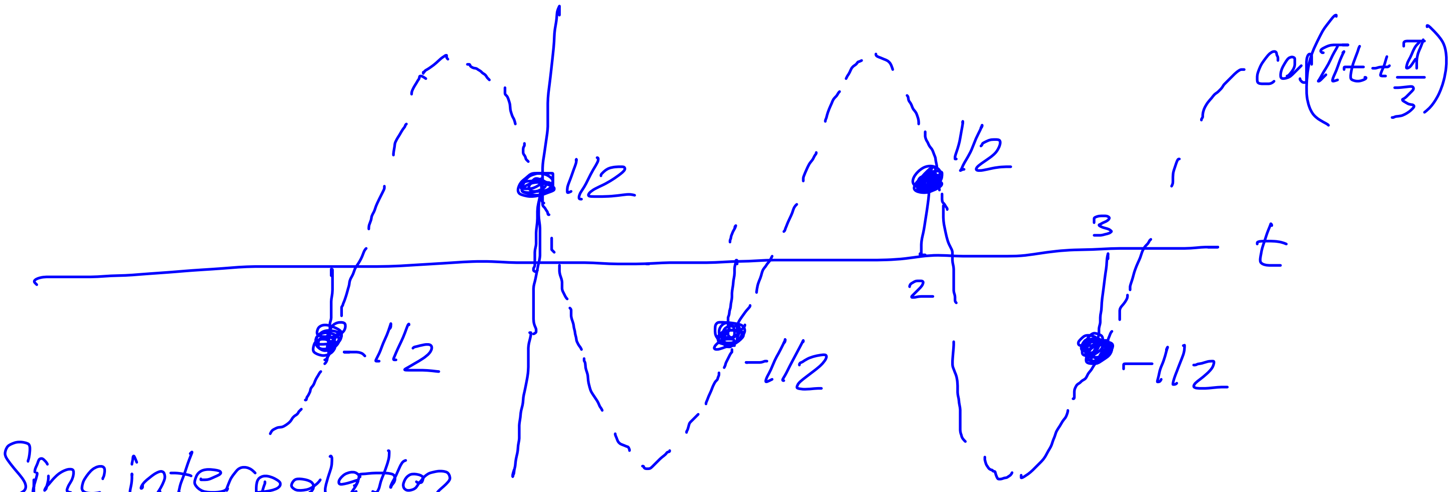
Same samples as part (c),
therefore sinc interpolation gives
the function in part (c.)

e) $\cos\left(\pi t + \frac{\pi}{3}\right)$ NO.

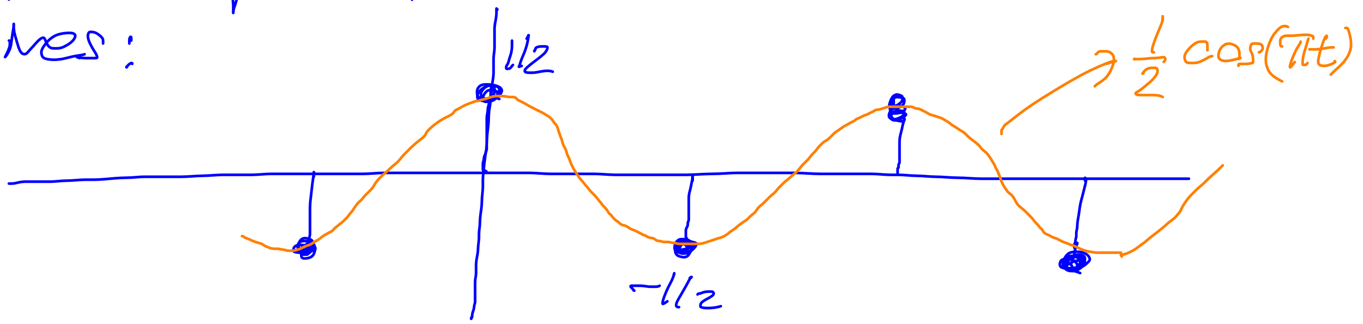
$\omega = \pi$ so we are at the boundary of the condition $\omega < \frac{\pi}{1} = \pi$.

The samples are

$$\begin{cases} \cos\frac{\pi}{3} = \frac{1}{2} & \text{if } t \text{ even} \\ \cos\frac{4\pi}{3} = -\frac{1}{2} & \text{if } t \text{ add.} \end{cases}$$



Sinc interpolation gives:



4. (15 points)

a) (6 points) Find a **real-valued** length-4 sequence $x(t)$, $t = 0, 1, 2, 3$, such that the DFT coefficients for $k = 1$ and $k = 2$ are

$$X(1) = 1 - j \quad X(2) = 0.$$

Is the answer unique? If not, give an example of another real-valued $x(t)$ with the same $X(1)$ and $X(2)$.

By conjugate symmetry $X(3) = X(1)^* = 1 + j$.

Then
$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix} = X(0)\vec{\Phi}_0 + \overbrace{X(1)}^{1-j}\vec{\Phi}_1 + \overset{=0}{\cancel{X(2)}}\vec{\Phi}_2 + \overbrace{X(3)}^{1+j}\vec{\Phi}_3$$

$$= \frac{X(0)}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{X(1)}{2}\begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix} + \frac{X(3)}{2}\begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix}$$

$$= \frac{X(0)}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1-j \\ j+1 \\ -1+j \\ -j-1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1+j \\ -j+1 \\ -1-j \\ j-1 \end{bmatrix}$$

$$= \frac{X(0)}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

The answer is not unique since $X(0)$ is free.

One possible answer for $x(t)$, $t = 0, 1, 2, 3$ is

$\{1, 1, -1, -1\}$ $(X(0) = 0)$

others include $\{2, 2, 0, 0\}$ $(X(0) = 2)$.

b) (5 points) Suppose a length-N sequence, where N is even, satisfies

$$x\left(t+\frac{N}{2}\right) = -x(t), \quad t = 0, 1, \ldots, \frac{N}{2}-1$$

that is, the second half of the sequence is the negative of the first half. Show that

$$X(k) = 0 \quad \text{when } k \text{ is even.}$$

Hint: First find a relation between W_k^t and $W_k^{t+\frac{N}{2}}$ where $W_k = e^{jk\frac{2\pi}{N}}$.

$X(k)$ is given by $\vec{\Phi}_k^{*} \vec{X}$ where

$$\Phi_k^{*} = \frac{1}{\sqrt{N}}\left[1 \quad W_k^{*} \quad ----- \quad W_k^{N-1*}\right], \qquad \vec{X} = \left[\begin{array}{c} x(0) \\ \vdots \\ x(\frac{N}{2}-1) \\ -x(0) \\ \vdots \\ -x(\frac{N}{2}-1) \end{array}\right] \begin{array}{l} \left.\vphantom{\begin{array}{c} x \\ \vdots \\ x \end{array}}\right\} \text{first half} \\ \left.\vphantom{\begin{array}{c} x \\ \vdots \\ x \end{array}}\right\} \text{second half} \end{array}$$

Then

$$X(k) = \frac{1}{\sqrt{N}}\Big(x(0) + x(1)W_k^{*} + --- + x(\tfrac{N}{2}-1)W_k^{\frac{N}{2}-1*} - x(0)W_k^{\frac{N}{2}+1*} - x(1)W_k^{\frac{N}{2}+2*} \; -- \; - x(\tfrac{N}{2}-1)W_k^{N-1*}\Big)$$

$$= \frac{1}{\sqrt{N}} \sum_{t=0}^{\frac{N}{2}-1} x(t) \underbrace{\left(W_k^t - W_k^{t+\frac{N}{2}}\right)^{*}}$$

this is zero when k is even because $W_k^{t+\frac{N}{2}} = W_k^t W_k^{N/2} = W_k^t e^{j\pi k}$

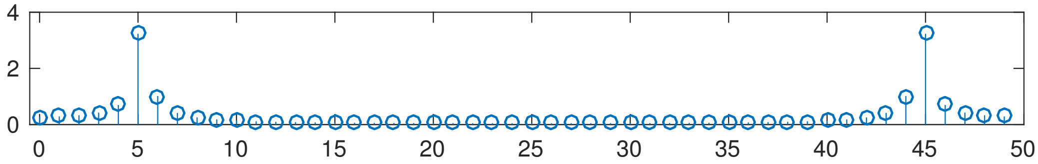
$= (-1)^k W_k^t = W_k^t$

Therefore $X(k) = 0$ when k is even.

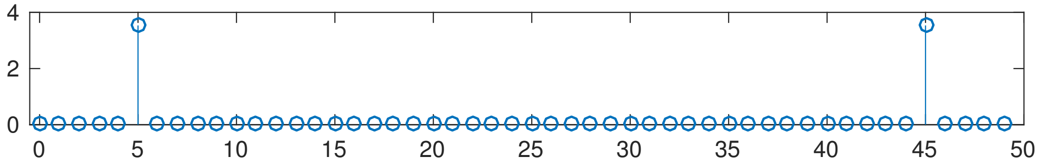
c) (4 points) Plots A-E show the magnitude of the DFT for the length-50 sequences below. Match each sequence to a DFT plot:

1 for all t → Plot C
$(-1)^t$ → Plot E
$\frac{1}{2}e^{j0.2\pi t}$ → Plot D
$\cos(0.2\pi t)$ → Plot B
$\cos(0.21\pi t)$ → Plot A

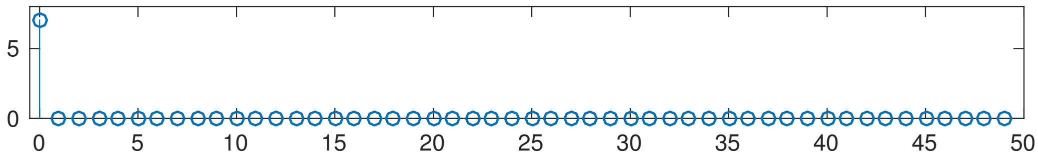
A



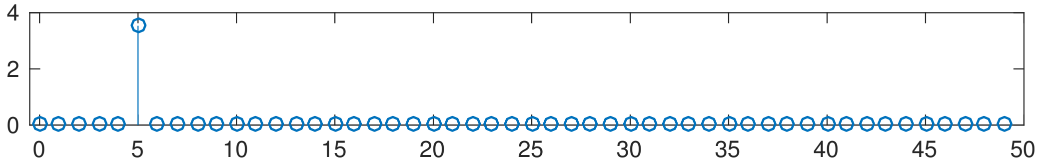
B



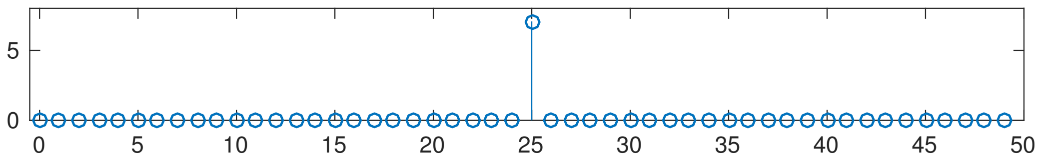
C



D



E



5. (10 points) Consider a system described by the input/output relation:

$$y(t) = a_0 u(t) + a_1 u(t-1) + \cdots + a_M u(t-M)$$

where M is a positive integer and $a_0, a_1, \ldots, a_M$ are constants.

a) (2 points) Determine if this system is linear and time-invariant. Explain your reasoning.

Linear: If u, y satisfy the relation above so do au, ay for any scaling factor a. If u_1, y_1 and u_2, y_2 satisfy the relation so does

$u_1 + u_2$, $y_1 + y_2$.

Time invariant because the coefficients $a_0, a_1, \ldots a_M$ are constant.

b) (2 points) Suppose the system above has impulse response

$$h(t) = \begin{cases} 0.5 & \text{when } t = 0 \\ -0.5 & \text{when } t = 1 \\ 0 & \text{otherwise.} \end{cases}$$

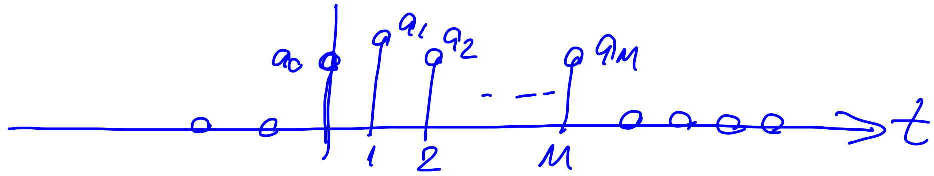
Determine the integer M and the coefficients $a_0, a_1, \ldots, a_M$.

The impulse response of system

$$y(t) = a_0 u(t) + a_1 u(t-1) + \cdots + a_M u(t-M)$$

is

$$h(t) = a_0 \delta(t) + a_1 \delta(t-1) + \cdots + a_M \delta(t-M)$$



Since we are given $h(0) = 0.5$, $h(1) = -0.5$ and $h(t) = 0$ for all other t, we conclude

$$\boxed{\begin{array}{l} M = 1 \\ a_0 = 0.5 \quad a_1 = -0.5 \end{array}}$$

c) (4 points) Find the DFT of $h(t)$, $t = 0, 1, \ldots, N-1$, where the length $N \geq 2$ is arbitrary. Your answer should provide a formula for $H(k)$, $k = 0, 1, \ldots, N-1$, that depends on k and the length N.

$$H(k) = \frac{1}{\sqrt{N}} \underbrace{\begin{bmatrix} 1 & W_k^* & - - - - \end{bmatrix}}_{\Phi_k^*} \underbrace{\begin{bmatrix} 0.5 \\ -0.5 \\ 0 \\ \vdots \end{bmatrix}}_{\vec{h}} \quad \text{where } W_k = e^{j\frac{2\pi}{N}k}$$

$$= \frac{1}{\sqrt{N}} 0.5 \left(1 - W_k^*\right)$$

$$\boxed{H(k) = \frac{1}{\sqrt{N}} 0.5 \left(1 - e^{-j\frac{2\pi}{N}k}\right)}$$

d) (2 points) Continuing part (c) now specify $H(k)$ when $k = 0$ and $k = N/2$ (assuming N is even). Which range of input frequencies (high or low) does this system suppress?

For $k=0$ $\quad e^{-j\frac{2\pi}{N}k} = 1$ regardless of N.

For $k=\frac{N}{2}$ $\quad e^{-j\frac{2\pi}{N}k} = e^{-j\pi} = -1.$

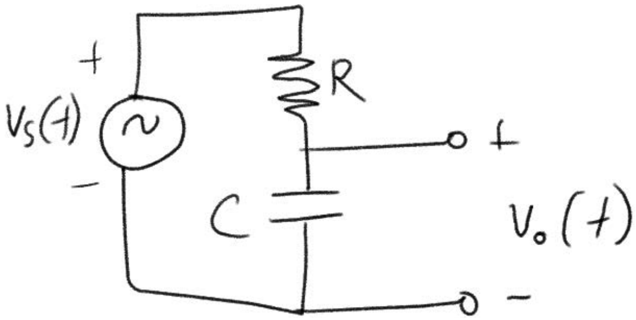
Therefore

$$H(0) = 0$$

$$H\left(\frac{N}{2}\right) = \frac{1}{\sqrt{N}}$$

and the low frequency $k=0$ is supressed.

6. (10 points) Consider the circuit below where $v_s(t)$ is a sinusoidal voltage at a single frequency, ω.



a) (5 points) Provide a symbolic expression for $v_0(t)$ as a function of $v_s(t)$.

$$V_s(t) = V_s \cos(\omega t + \phi_o)$$

+1 for $v_s(t)$ explicit or implicit

+1 for $\tilde{H}(\omega)$ expression

$$\tilde{V}_o = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \tilde{V}_s$$

$$= \frac{1}{1 + j\omega RC} \tilde{V}_s$$

$$\tilde{H}(\omega) = \frac{\tilde{V}_o}{\tilde{V}_s}$$

$$|\tilde{H}| = \frac{1}{|1 + j\omega RC|} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\phi_{\tilde{H}} = -\tan^{-1}(\omega RC)$$

+2 for proper $|H|$ and ϕ

$$\boxed{V_o(t) = V_s |\tilde{H}| \cos(\omega t + \phi_o + \phi_{\tilde{H}})}$$

+1 for proper time domain expression

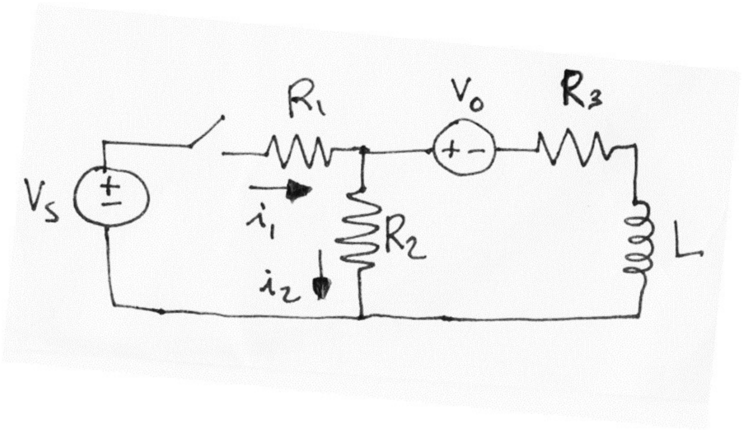
b) (5 points) If $C = 1\mu F$ and $\omega = 1$ rad/s find R such that the output is phase shifted by $\pi/4$ from the input.

$$\phi_{\tilde{H}} = -\tan^{-1}(\omega RC) = -\frac{\pi}{4} \leftarrow +3 \text{ for proper condition}$$

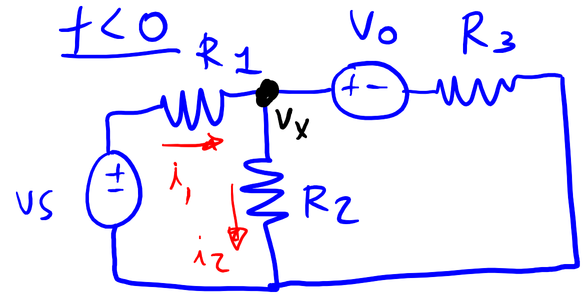
$$\tan^{-1}(\omega RC) = \frac{\pi}{4}$$

$$\left.\begin{aligned} \omega RC &= 1 \\ (1\ \text{rad/s})(10^{-6} F)\, R &= 1 \\ R &= 1\, M\Omega \end{aligned}\right\} +2 \text{ for correct math}$$

7. (10 points) Consider the circuit below. The switch is closed until $t = 0$ s, then opened. V_s, V_0, R_1, R_2, R_3, and L are given.



a) (2.5 points) Provide a symbolic expression for i_1 at $t < 0$ s.



$$\frac{V_x - V_s}{R_1} + \frac{V_x}{R_2} + \frac{(V_x - V_0)}{R_3} = 0$$

$$\frac{V_x}{R_1} - \frac{V_s}{R_1} + \frac{V_x}{R_2} + \frac{V_x}{R_3} - \frac{V_0}{R_3} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_x = \frac{1}{R_1}V_s + \frac{1}{R_3}V_0$$

$$\boxed{V_x = \left(\frac{1}{R_1}V_s + \frac{1}{R_3}V_0\right)(R_1 \| R_2 \| R_3)}$$

$$\boxed{i_1 = \frac{V_s - V_x}{R_1}}$$

+1.5 for proper method to solve for v_x or i_1

+ 1 for correct math

(I am not too concerned if they simplify or not).

b) (2.5 points) Provide a symbolic expression for i_2 at $t < 0$ s.

$$i_2 = \frac{v_x}{R_2}$$

same as above

re: simplification

c) (5 points) Provide a symbolic expression for i_2 at $t \geq 0$ s.

$$i_L(t \leq 0) = \frac{V_x - V_0}{R_3}$$

+1 for $i_L(0)$

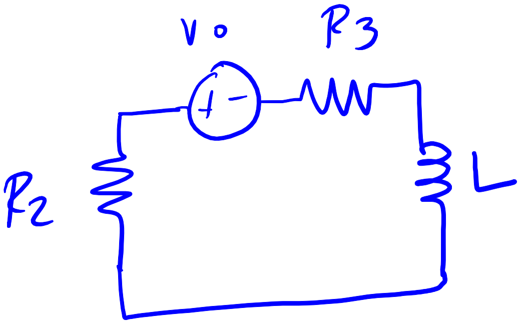
+1 for $i_L(\infty)$

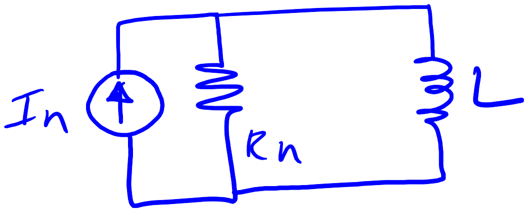
+1 for circuit

+1 for ODE

+1 for solution

$\underline{t \geq 0}$





$$I_n = -\frac{V_0}{R_n} \qquad R_n = R_2 + R_3$$

$$i_L(t) = i_L(\infty) + \left(i(0) - i(\infty)\right)e^{-\frac{R_n}{L}t}$$

$$\boxed{i_L(t) = I_n + \left[\frac{V_x - V_0}{R_3} - I_n\right] e^{-\frac{R_n}{L}t}}$$

$$i_L(t \to \infty) = I_n$$

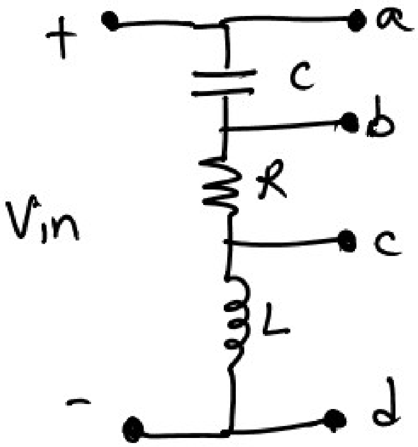
$$-I_n + \frac{V_x}{R_n} + i_L = 0$$

$$-I_n + \frac{L}{R_n}\frac{di}{dt} + i_L = 0$$

$$\frac{di}{dt} + \frac{R_n}{L} i_L = \frac{R_n}{L} I_n$$

8. (10 points) Consider the circuit below.

Rubric for this problem is rather black/white.



a) (2.5 points) Across which two output terminals would the voltage transfer function look like a low pass filter with respect to V_{in}?

a an b (capacitor)

b) (2.5 points) Write the transfer function for your answer in (a).

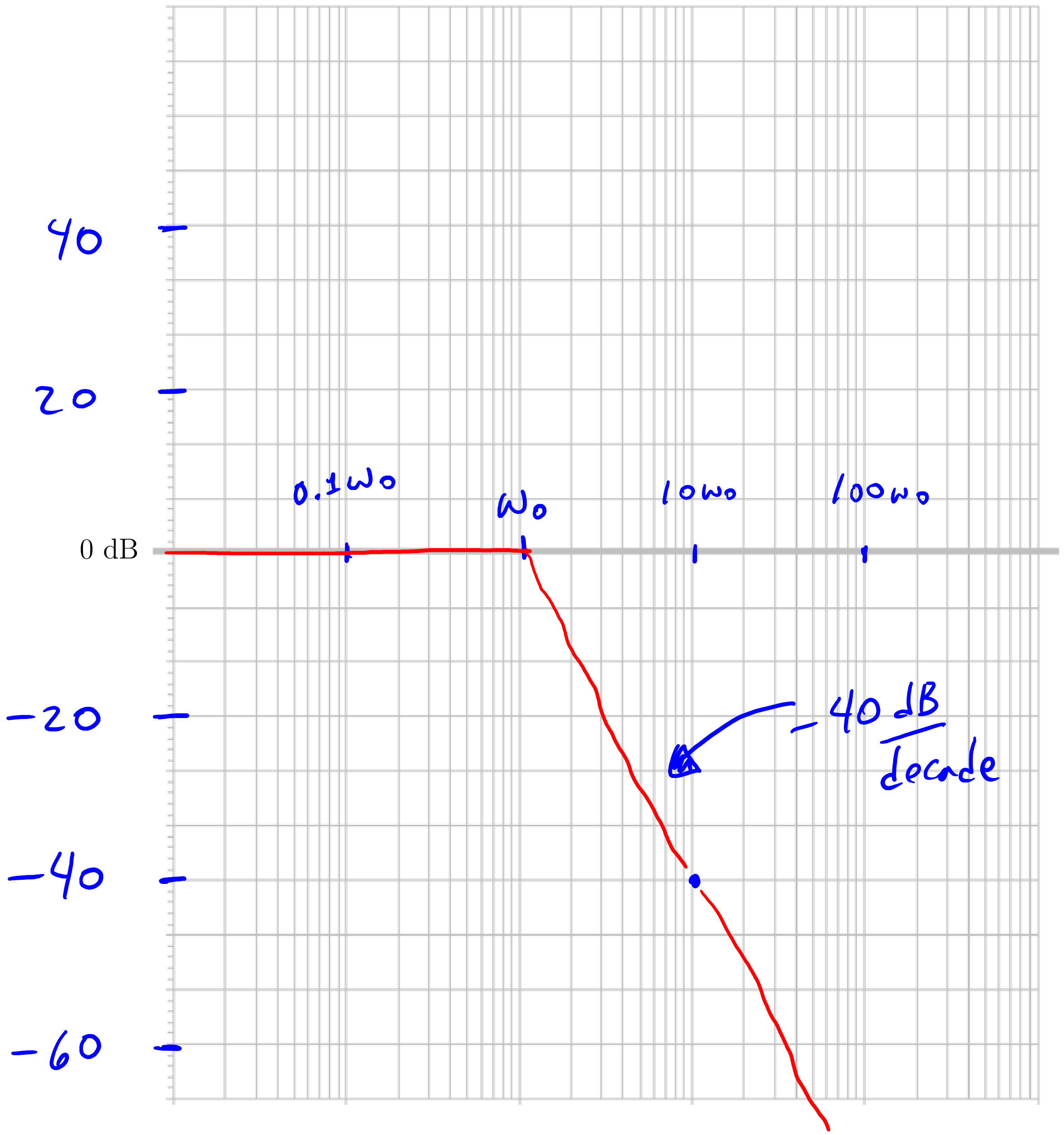
$$\tilde{H}(\omega) = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC + (j\omega L)j\omega C}$$

$$= \frac{1}{1 + j\omega RC - \left(\frac{\omega}{\omega_0}\right)^2} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

This is a quadratic pole at ω_0.

+1 if they point this out (not required) not full freq. response!

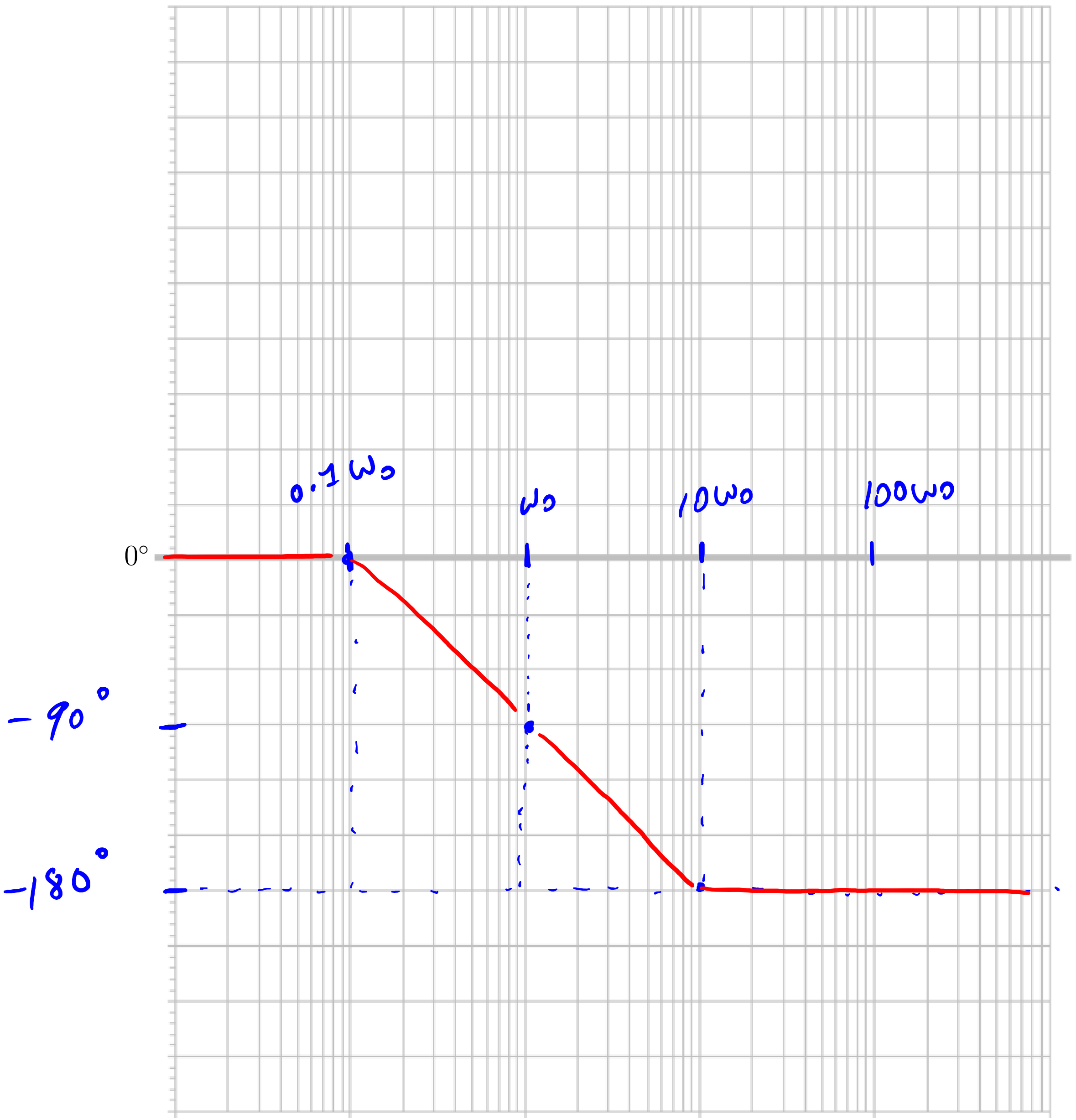
c) (2.5 points) Plot the magnitude Bode plot of the transfer function. (You can use the table on page 32.)



+1.5 for correct form

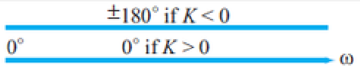
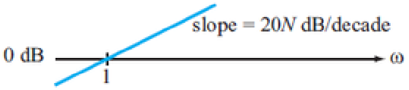
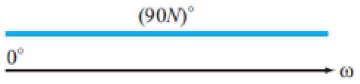
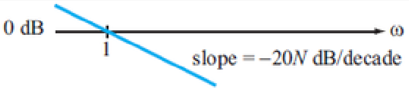
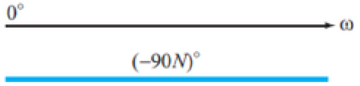
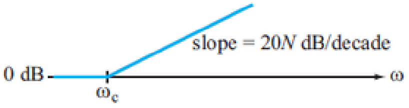
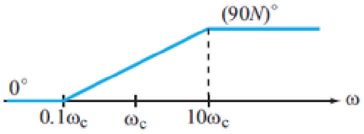
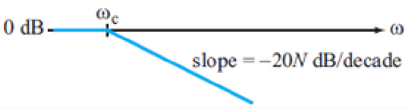
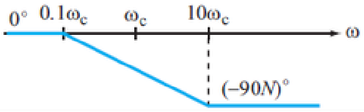
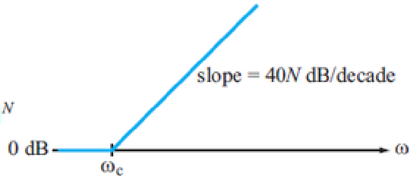
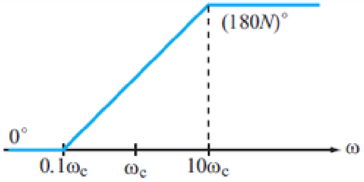
+1 for proper slope and low pass level (0dB)

d) (2.5 points) Plot the phase Bode plot of the transfer function. (You can use the table on page 32.)



Additional workspace for Problems 8c and 8d.

Table 9-2: Bode straight-line approximations for magnitude and phase.

Factor	**Bode Magnitude**	**Bode Phase**
Constant K		
Zero @ Origin $(j\omega)^N$		
Pole @ Origin $(j\omega)^{-N}$		
Simple Zero $(1 + j\omega/\omega_c)^N$		
Simple Pole $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$		
Quadratic Zero $[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N$		
Quadratic Pole $\frac{1}{[1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2]^N}$