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EECS 16B: FALL 2015—FINAL

Important notes: Please read every question carefully and completely – the setup may or may not be the same as what you have seen before. Also, be sure to show your work since that is the only way we can potentially give you partial credit.

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Problem 2: _____/ 20

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Total: _____/ 129

PROBLEM 1. Miscellaneous (20 pts)

a) **(6 pts)** If x[n] = $e^{j(2\pi/3)n} + e^{j(2\pi/5)n}$, compute the coefficients X[k] for a length-15 DFT of the signal over the interval n = 0, 1, … , 14.

* Length 15 contains integer multiple of periods of both components, so dont need to worry about spectral leakage.

* $X[k] = \sum_{n=0}^{14} e^{-j\left(\frac{2\pi}{15}\right)kn} \cdot \left(e^{j\left(\frac{2\pi}{3}\right)n} + e^{j\left(\frac{2\pi}{5}\right)n}\right)$

$\neq 0$ only when $\frac{2\pi}{15}kn = \frac{2\pi}{3}n$ or $\frac{2\pi}{15}kn = \frac{2\pi}{5}n$

$k = 5$ $\qquad$ $k = 3$

* So, $X[3] = X[5] = 15$

b) **(4 pts)** Many bio-sensors are constructed by getting the molecule of interest to attach to a magnetic bead, and then subsequently having that bead become chemically/physically attached (usually via specific antibodies) to an inductor. The number of such attached molecules is then sensed by measuring the value of the inductor.

Assuming that our sense inductor is a solenoid with an area A = 1e-7 m^2, number of turns N = 4, and length l = 10μm, if each magnetic bead that gets attached to the inductive sensor adds 4π∗1e-10 to the permeability (the permeability with zero beads is 4π∗1e-7), provide an expression for the inductance of the sensor as a function of the number of beads N_{beads}.

$$L_{sense} = \mu_{eff} \frac{N^2 A}{l} = \left(4\pi e\text{-}7 + N_{beads} \cdot 4\pi e\text{-}10\right) \frac{H}{m} \cdot \frac{4^2 \cdot 1e\text{-}7\ m^2}{10e\text{-}6\ m}$$

$$= 4\pi e\text{-}7 \cdot 16e\text{-}2 \cdot \left(1 + 1e\text{-}3 \cdot N_{beads}\right) H$$

$$= 64nH \cdot \left(1 + 1e\text{-}3 \cdot N_{beads}\right)$$

c) **(10 pts)** For this problem, assume you have an open-loop system whose dynamics are described by:

$$\begin{bmatrix} x_0[k+1] \\ x_1[k+1] \\ x_2[k+1] \end{bmatrix} = A \begin{bmatrix} x_0[k] \\ x_1[k] \\ x_2[k] \end{bmatrix} + Bu[k] \qquad y[k] = C \begin{bmatrix} x_0[k] \\ x_1[k] \\ x_2[k] \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If we place this system in to closed-loop feedback with a forward gain $K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$, choose values for k_1, k_2, and k_3 such that the eigenvalues of the closed-loop system are equal to 0.9, 0.8, and 0.7.

Important Note: You do not need to know the formula for the determinant of a 3x3 matrix to solve this problem. Instead, recall that for an eigenvector $v = [v_0\ v_1\ v_2]^T$ (where you can initially treat e_0, e_1, and e_2 as being unknowns), $Av = \lambda v$, and use this relationship to determine an equation for λ.

$$A_{CL} = A - B\underline{K}C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-k_1 & 2-k_2 & 3-k_3 \end{bmatrix}$$

Let's define this as: $A_{CL} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c_0 & c_1 & c_2 \end{bmatrix}$

$A_{CL} v = \lambda v \longrightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c_0 & c_1 & c_2 \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \lambda v_0 \\ \lambda v_1 \\ \lambda v_2 \end{bmatrix} \longrightarrow$

$v_1 = \lambda v_0$

$v_2 = \lambda v_1 = \lambda^2 v_0$

$c_0 v_0 + c_1 v_1 + c_2 v_2 = \lambda v_2 \longrightarrow c_0 v_0 + c_1 \lambda v_0 + c_2 \lambda^2 v_0 = \lambda^3 v_0$

$\lambda^3 - c_2 \lambda^2 - c_1 \lambda - c_0 = 0$

Desired evals: $(\lambda - 0.9)(\lambda - 0.8)(\lambda - 0.7) = 0$

$\lambda^3 + (-0.7 - 0.8 - 0.9)\lambda^2 + (0.9 \cdot 0.8 + 0.9 \cdot 0.7 + 0.8 \cdot 0.7)\lambda - 0.9 \cdot 0.8 \cdot 0.7 = 0$

$\lambda^3 - 2.4\lambda^2 + 1.91\lambda - 0.504 = 0$

So: $-c_2 = -2.4 \rightarrow k_3 = 0.6$

$-c_1 = 1.91 \rightarrow k_2 = 3.91$

$-c_0 = -0.504 \rightarrow k_1 = 0.496$

PROBLEM 2. PI control (20 points)

(a) **(3 pts)** Let's assume that we have an open-loop system whose dynamics are described by:

$$x_0[k+1] = Ax_0[k] + Bu[k]$$

$$y[k] = Cx_0[k]$$

$$A = 0.8, B = 1, C = 0.5$$

Assuming we have a desired output $y_d[k]$ and that we close the above system in a feedback loop with a forward gain K, derive the A_{CL}, B_{CL}, and C_{CL} that capture the dynamics of the resulting closed-loop system.

$A_{CL} = A - BKC = 0.8 - 1 \cdot K \cdot 0.5$ $\quad$ $B_{CL} = BK$ $\quad$ $C_{CL} = 0.5$

$= 0.8 - 0.5K$ $\quad$ $= K$

(b) **(4 pts)** If $K = 2$ and $y_d = 2$ (i.e., y_d is a constant), what will $y[k]$ converge to? In other words, what will be the steady state error between y_d and y?

$x[k] = A_{CL}\, x[k] + B_{CL}\, y_d[k]$

$(1 - 0.8 + 0.5K)\, x[k] = K y_d[k]$

$x[k] = \frac{K}{0.2 + 0.5K} \cdot y_d$ $\quad$ $K = 2 \rightarrow x[k] = \frac{2}{1.2} \cdot 2$

$y_d = 2$ $\quad$ $x[k] = \frac{40}{12} = \frac{10}{3}$

$y = 0.5x \rightarrow$ error $= y - y_d = \frac{5}{3} - 2 = -\frac{1}{3}$

(c) **(8 pts)** Now let's assume that we augment the system to include an integral term in the feedback. However, unlike the examples we developed in lecture, we will add this integral feedback to our original proportional-only controller. In other words, the output of our controller $u[k]$ is now set by:

$$u[k] = K_P(y_d[k] - y[k]) + z[k]$$
$$z\left[k+1\right] = K_I \, \mathrm{T}_s(y_d[k] - y[k]) + z[k]$$

Re-derive the A_{CL}, B_{CL}, and C_{CL} that capture the dynamics of the resulting closed-loop system. Hint: what are the state variables of the new closed-loop system?

We'll need to include $z[k]$ as a state variable, but let's first plug in the controller output in to the open-loop system:

$$x[k+1] = Ax[k] + BK_p(y_d[k] - y[k]) + Bz[k]$$
$$= Ax[k] + BK_p(y_d[k] - Cx[k]) + Bz[k]$$
$$= (A - BK_pC)x[k] + Bz[k] + BK_p y_d[k]$$

$$z[k+1] = K_I T_s (y_d[k] - y[k]) + z[k]$$
$$= K_I T_s (y_d[k] - Cx[k]) + z[k]$$
$$= -K_I T_s C x[k] + z[k] + K_I T_s y_d[k]$$

$$\vec{x}[k] = \begin{bmatrix} x[k] \\ z[k] \end{bmatrix}$$

So: $A_{CL} = \begin{bmatrix} A - BK_pC & B \\ -K_I T_s C & 1 \end{bmatrix}$ $\quad B_{CL} = \begin{bmatrix} BK_p \\ K_I T_s \end{bmatrix}$ $\quad C_{CL} = [0.5 \quad 0]$

$= \begin{bmatrix} 0.8 - 0.5K_p & 1 \\ -K_I T_s/2 & 1 \end{bmatrix}$ $\quad = \begin{bmatrix} K_p \\ K_I T_s \end{bmatrix}$

(d) **(5 pts)** As a function of K_I and K_P but continuing to assume that $y_d = 2$ (i.e., y_d is a constant), what will $y[k]$ converge to with the new feedback controller from part (c)? Note that you must derive the answer – you can't simply state the final result.

$$\begin{bmatrix} x[k] \\ z[k] \end{bmatrix} = \begin{bmatrix} 0.8-0.5K_p & 1 \\ -K_I T_s/2 & 1 \end{bmatrix} \begin{bmatrix} x[k] \\ z[k] \end{bmatrix} + \begin{bmatrix} K_p \\ K_I T_s \end{bmatrix} y_d[k]$$

$$\begin{bmatrix} 0.2+0.5K_p & -1 \\ K_I T_s/2 & 0 \end{bmatrix} \begin{bmatrix} x[k] \\ z[k] \end{bmatrix} = \begin{bmatrix} K_p \\ K_I T_s \end{bmatrix} y_d[k]$$

$$K_I T_s \cdot \tfrac{1}{2} x[k] = K_I T_s y_d[k]$$

$$y[k] = C x[k] = \tfrac{1}{2} x[k]$$

$$K_I T_s y[k] = K_I T_s y_d[k]$$

$$\hookrightarrow y[k] - y_d[k] = 0$$

PROBLEM 3. DC-DC Converters (27 pts)

Having learned about Dynamic Voltage and Frequency Scaling (DVFS) in lecture, you may now have an appreciation for why it would be useful to have a circuit component that can take a battery input voltage (e.g., ~3.7V for a lithium-ion battery) and efficiently (i.e., ideally, without the circuit itself dissipating any power) generating a new power supply voltage with an arbitrary value (e.g., 0.5V – 1V to supply a digital processor, or e.g. >5-10V to make the LCD work). These circuits are known as "DC-DC converters", and in this problem we will explore some key sub-components to get an idea of how such circuits operate.

Important note: This problem has many sub-parts, but has been set up so that the majority of the sub-parts can be solved independently of each other.

a) **(5 pts)** Let's first remind ourselves of the benefits of being able to scale the supply voltage of a digital circuit. Assuming that with a supply voltage V_{dd} = 1V our digital circuit operates at 1GHz and dissipates 1W, how much power will the digital circuit dissipate at a supply voltage of 0.6V and clock frequency of 300MHz?

$$P = \alpha_{0\to 1} C V_{DD}^2 f$$

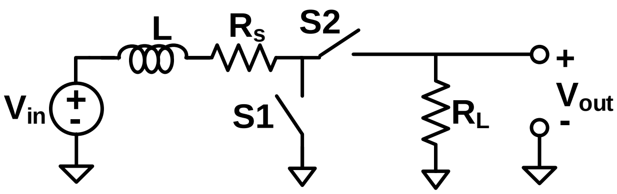
$$P_{new} = \left(\frac{V_{DD,new}}{V_{DD,old}}\right)^2 \cdot \left(\frac{f_{new}}{f_{old}}\right) \cdot P_{old}$$

$$= \left(\frac{0.6V}{1V}\right)^2 \cdot \left(\frac{300MHz}{1GHz}\right) \cdot 1W$$

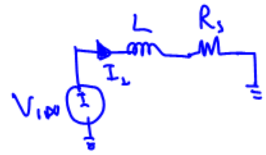
$$= \boxed{108\,mW}$$

b) **(10 pts)** For the circuit shown below, assuming that for t<0 switch S1 is on and switch S2 if off, while for t≥0 switch S1 is off and switch S2 is on, derive an expression for $V_{out}(t)$ for t≥0.

(This isn't important to solving this problem, but note that R_L here represents the circuit that we are trying to power through the hypothetical DC-DC converter.)



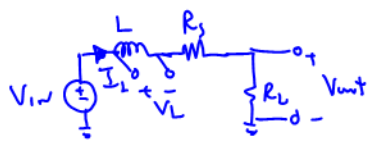
When S1 is on:



Since we are in this state for all negative time, we can assume everything has settled, and so

$I_L(0) = \frac{V_{IN}}{R_S}$

When S2 turns on:



$V_{IN} - L\frac{dI_L}{dt} - I_L(R_S+R_L) = 0$

$V_{IN} - (R_S+R_L)I_L = L\frac{dI_L}{dt}$

$\frac{V_{IN}}{R_S+R_L} - I_L = \frac{L}{R_S+R_L}\cdot\frac{dI_L}{dt}$

$I_L' = \frac{-L}{R_S+R_L}\cdot\frac{dI_L'}{dt}$

$I_L'(t) = I_L'(0)\,e^{-\frac{R_S+R_L}{L}\cdot t}$

$I_L'(0) = \frac{V_{IN}}{R_S+R_L} - I_L(0)$, $I_L(0) = \frac{V_{IN}}{R_S}$

$I_L'(0) = \frac{R_S - R_S - R_L}{R_S(R_S+R_L)}\cdot V_{IN} = -\frac{R_L}{R_S+R_L}\frac{V_{IN}}{R_S}$

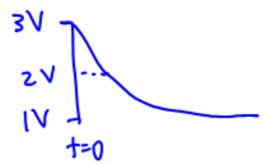
$I_L(t) = \frac{V_{IN}}{R_S+R_L} - I_L'(t) = \frac{V_{IN}}{R_S+R_L} + \frac{R_L}{R_S+R_L}\cdot\frac{V_{IN}}{R_S}\cdot e^{-\frac{R_S+R_L}{L}\cdot t}$

$V_{out}(t) = R_L\cdot I_L(t) = \boxed{\frac{R_L}{R_S+R_L}\left(1+\frac{R_L}{R_S}\cdot e^{-\frac{R_S+R_L}{L}\cdot t}\right)\cdot V_{IN}}$

c) **(4 pts)** Assuming that after plugging in values for the various circuit components your answer to part b) was that $V_{out}(t) = 1V + 2V * e^{(-t/500ns)}$, how long after changing the state of the switches (at t=0) will it take for $V_{out}(t)$ to reach 2V?

(This isn't important to solving the problem, but for the type of DC-DC converter being modeled here, the circuit is typically operated such that switches S1 and S2 toggle states at a period faster than the time you computed here.)

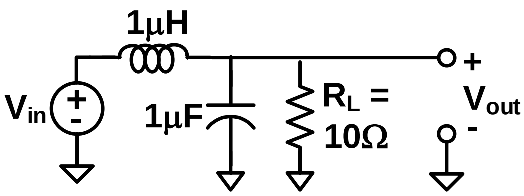
Sketch waveform:



2V is "half way", so can just use "delay" formula:

$\boxed{t_{delay} = \ln(2) \cdot 500ns}$

d) **(8 pts)** Now let's look at the key components of another type of DC-DC converter shown below and known as a "buck converter". Assuming that $V_{in}(t)$ is a sinusoid, write V_{out} as a function of V_{in} and the component values provided.



Use impedances: $\quad R_L \,||\, Z_{1\mu F} = \dfrac{10 \cdot \frac{1}{j\omega\, 1e\text{-}6}}{10 + \frac{1}{j\omega\, 1e\text{-}6}} = \dfrac{10}{1 + j\omega\, 10e\text{-}6}$

$$\frac{V_{out}}{V_{in}} = \frac{10/(1+j\omega\, 10e\text{-}6)}{10/(1+j\omega\, 10e\text{-}6) + j\omega\, 1e\text{-}6} = \frac{10}{10 + j\omega\, 1e\text{-}6 - \omega^2\, 10e\text{-}12}$$

$$= \boxed{\frac{1}{1 - \omega^2 \cdot 1e\text{-}12 + j\omega \cdot 1e\text{-}7}}$$

e) **(BONUS: 5 pts)** Assuming that $V_{in}(t) = V_{DC} + 2*V_{DC}*\cos(2\pi*10MHz*t)$ and given your answer to part d), provide an approximate expression for $V_{out}(t)$.

At DC $w=0$, so $\frac{V_{out}}{V_{in}} = \frac{1}{1-0+j0} = 1$

At 10MHz, $\left\|\frac{V_{out}}{V_{in}}\right\| = \frac{1}{\sqrt{(1-(2\pi\cdot 1e7)^2\cdot 1e\text{-}12)^2 + (2\pi\cdot 1e7\cdot 1e\text{-}6)^2}}$

$\approx \frac{1}{3947}$

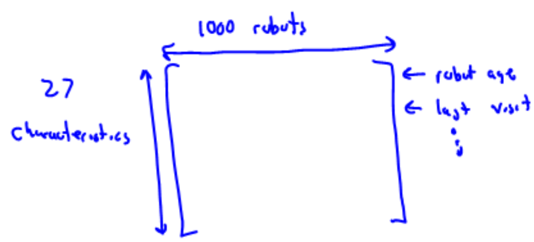
So, the AC component is almost completely killed by the LRC network, and hence $V_{out}(t) \approx V_{DC}$

PROBLEM 4. Robots Healing the World (18 points)

After successfully taking its start-up for robot bicycles public, our enterprising robot SixT33n from HW6 decides it is time to turn its attention to more “humanitarian” endeavors. So, SixT33n teams up with Dr. MD from Grey Hospital in Seattle and tries to help solve some of the hospital’s most pressing problems.

a) **(4 pts)** Grey Hospital doesn’t only treat humans, but robots like SixT33n as well. The (human) doctors at the hospital however need some help in making quick diagnoses of what might be ailing the robot. It turns out that almost all of the robots that come to Grey Hospital suffer from one of three “medical” conditions. To help address the need for fast diagnosis of which condition each robot might be suffering from, SixT33n decides to gather 27 pieces of characteristic data (e.g., how old the robot is, when it last visited the hospital, how many times per week it oils its joints, etc.) from each of the robots that comes to Grey Hospital, as well as recording which (if any) of the three conditions the robots suffer from.

After having collected data from 1000 robot visitors, explain how SixT33n could set up a matrix *A* that could be analyzed to diagnose future robots. You should arrange *A* such that the information from each robot is a column in the matrix. Be sure to indicate what the dimensions of the matrix *A* are.



b) **(8 pts)** Assuming that the *A* matrix has two very dominant singular values, for any new robot visitor *i* whose data is collected, explain how SixT33n should use the matrix *A* and the new visitor's individual data vector a_i to make a diagnosis.

* First use SVD to decompose A in to $U \Sigma V^T$

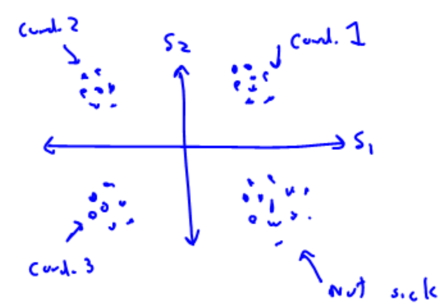
* Know that have two principal components since have two dominant singular values, so probably want to use both of the associated singular vectors.

* Note that this time characteristics are arranged in the column, so need to use first two left singular vectors u_1 & u_2 for projection.

* For each new robot a_i, form a vector comprised of $\begin{bmatrix} a_i^T u_1 \\ a_i^T u_2 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$

* Plot this vector on a plane & check which cluster it is closest to in order to make the diagnosis

E.g.



c) **(6 pts)** Based only on the originally collected data in the *A* matrix and the recordings of which of those robots suffered from which conditions, and still assuming the *A* matrix has two very dominant singular values, explain how you might predict the accuracy with which SixT33n can diagnose a new visitor.

* Accuracy can be predicted by examining with the selected boundaries between the clusters (conditions), what percentage of the cases contained within the A matrix fall in to the correct cluster.

* Note that of course still using the same two first left singular vectors to create the two dimensional score $[s_1 \; s_2]^T$

PROBLEM 5. Robots Might Cause Trouble Too (24 points)

SixT33n's diagnosis method from Problem 2 turns out to be wildly successful and robots from all over Seattle are now flooding to Grey Hospital. Unfortunately however, when enough robots are in GSM Hospital all at once, some of the equipment for humans starts failing – i.e., giving false readings. In fact, an important piece of monitoring equipment fails right when Dr. MD is not only in the middle of a brain surgery, but one that is being recorded for live TV, leading to some very bad publicity.

SixT33n is once again called in to save the day, and quickly realizes that all of the equipment that is failing was designed before robots became prevalent. Specifically, the equipment samples its (electrical) input signals at 100kHz (i.e., T_s = 10μs), but because of their underlying circuitry, robots generate (and radiate) significant interfering signals at 80kHz. Even though the front-end of the equipment includes an anti-aliasing filter, when too many robots are present, the interfering signal becomes strong enough that after aliasing the errors are large enough to make the equipment fail.

a) **(4 pts)** After sampling, what continuous-time frequency (in Hz) will the interfering signals generated by the robots be indistinguishable from?

Assume $V_{int}(t) = e^{j(2\pi\, 80kHz)t}$

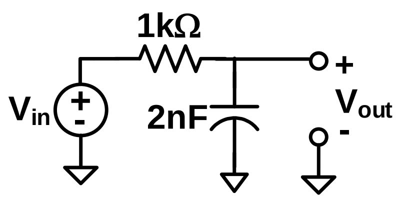
$$V_{int,d}[n] = e^{j\,2\pi\, 80kHz \cdot \frac{n}{100kHz}}$$

$$= e^{j2\pi \cdot 0.8 \cdot n}$$

$$= e^{j2\pi(0.8-1)\cdot n} = e^{j2\pi(-0.2)n}$$

So equivalent to continuous time f of $-20kHz$.

b) **(8 pts)** Assuming that the existing anti-aliasing filter within the equipment is implemented as shown below, if each robot generates an interfering signal $V_{int}(t) = 100\mu V * \cos(2\pi * 80kHz * t)$ in to the anti-aliasing filter, and that the interference from each robot is added together (i.e., with two robots, the amplitude of the interference is 200μV, with 3 it is 300μV, etc.), how many robots can be present before the magnitude of the interference after the anti-aliasing filter is >1mV (causing the equipment to fail)?



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega\, 2e\text{-}9 \cdot 1e3} = \frac{1}{1 + j\omega\, 2e\text{-}6}$$

$$\left\| \frac{V_{out}}{V_{in}} \right\|_{\omega = 2\pi \cdot 80kHz} = \frac{1}{\sqrt{1 + (2\pi \cdot 80e3 \cdot 2e\text{-}6)^2}}$$

$$\approx 0.7$$

So, $\|V_{out}\|$ due to interference $= 0.7 \cdot 100\mu V \cdot N_{robots}$

$$0.7 \cdot 100\mu V \cdot N_{robots} > 1mV$$

$\boxed{N_{robots} \geq 15}$ will cause the equipment to fail

c) **(12 pts)** In order to try and help alleviate the problem, SixT33n decides to add another analog circuit in front of the existing front-end to further reduce the interference. After adding this extra circuit, the equipment must be able to receive desired signals at frequencies of up to 40kHz without attenuating them by more than 25% (i.e., $V_{out}/V_{in} > 0.75$ for signals at this frequency). Similarly, when combined with the existing anti-aliasing filter, your design must attenuate the interference by at least a factor of 2 (i.e., $V_{out}/V_{in} < 0.5$ for signals at this frequency).

Using any combination of components you would like, design an analog circuit that meets these constraints. Be sure to label the values of any passive components (R's, C's, L's, etc.) that you use for this design.

To make our lives simple, let's assume that we just add another (cascaded) independent filter so that the overall response is:

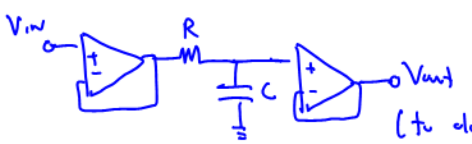
$$H_{total}(\omega) = H_{new\_filter}(\omega)\cdot H_{old\_filter}(\omega)$$

Let's first try a first-order $H_{new\_filter}(\omega)$; we already know that at the interfering f $\|H_{old\_filter}\| \approx 0.7$, so:

$$H_{new\_filter} = \frac{1}{1+j\omega\cdot\tau_c}$$

$$\|H_{new\_filter}\|\cdot\|H_{old\_filter}\| < 0.5$$

$$\frac{1}{\sqrt{1+(2\pi 80e3\cdot\tau_c)^2}} < \frac{0.5}{0.7}$$



$$\frac{1}{1+(2\pi 80e3\cdot\tau_c)^2} < \frac{25}{49}$$

$$\frac{49}{25} < 1+(2\pi 80e3\cdot\tau_c)^2$$

$$\frac{\sqrt{24}}{5} < 2\pi 80e3\cdot\tau_c$$

$$\tau_c > \frac{\sqrt{24}}{5}\cdot\frac{1}{2\pi 80e3}$$

Choose $\tau_c = \frac{1}{2\pi 80e3}$

Check in-band spec:

$$\underbrace{\frac{1}{\sqrt{1+\left(\frac{2\pi 40e3}{2\pi 80e3}\right)^2}}}_{H_{new}}\cdot\underbrace{\frac{1}{\sqrt{1+(2\pi 40e3\cdot 2e-6)^2}}}_{H_{old}}$$

$\approx 0.8 > 0.75$ ✓

$R = 1k\Omega$

$$C = \frac{\tau_c}{R} = \frac{1}{2\pi 80e6} \approx 1.99nF$$

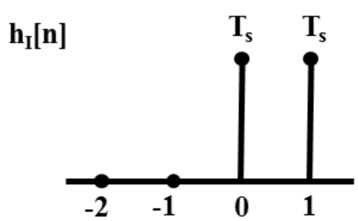
(2nF works too actually)

PROBLEM 6. Frequency-Domain in Control (20 points)

Although in this class we studied exclusively time-domain methods to analyze and design control systems, in future courses you will find that frequency-domain methods are extremely common when designing control systems as well. In this problem we will explore some of basic relationships between the time-domain and frequency-domain views to gain some preliminary insights.

Throughout this problem, you should recall (from both EE16A and EE16B) that the frequency response of a discrete-time time filter can be found by measuring its response to a complex exponential input, and that this frequency response will be related to the DFT of the impulse response of the filter.

a) **(8 pts)** Given the (truncated) impulse response of an integral controller shown below $h_I[n]$, compute the coefficients $H_I[k]$ for a length-4 DFT of $h_I[n]$ using the same window indicated in the impulse response.



$$H_I[k] = \sum_{n=-2}^{1} h_I[n]\cdot e^{-j\left(\frac{2\pi}{4}\right)kn}$$

$$H_I[0] = \sum h_I[n] = 2T_s$$

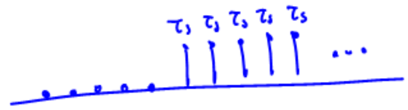
$$H_I[1] = T_s\left(1+e^{-j\frac{\pi}{2}}\right) = T_s(1-j)$$

$$H_I[2] = T_s(1+-1) = 0$$

$$H_I[3] = T_s\left(1+e^{-j\frac{3\pi}{2}}\right) = T_s(1+j)$$

b) **(3 pts)** If we were to take an infinitely long interval for both the impulse response and the DFT of the integral controller, what value would $H_I[0]$ (i.e., the DC gain) of the controller converge to?

As interval increases, impulse response looks like this



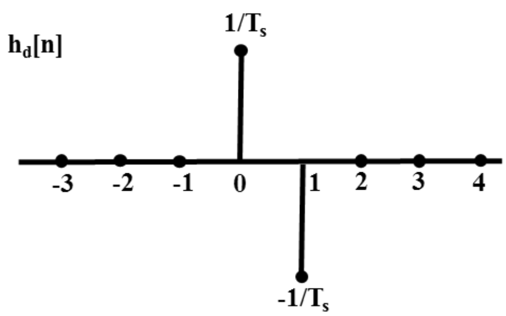
Since $H_I[0] = \sum h[n]$, with infinite n we will get that

$\boxed{H_I[0] = \infty}$

c) **(3 pts)** Given your answer to part b), why does an integral controller help reduce steady-state error?

∞ gain at DC means that any steady-state error would get multiplied by ∞ gain to counter it in the feedback – i.e., the error should go to zero (just like with ideal op-amps in feedback)

d) **(6 pts)** Now let's consider the impulse response of a derivative controller as shown below. If the DFT coefficients associated with this impulse response are $H_D[k]$, regardless of the number of points we used in the impulse response and the corresponding DFT, for what frequency (i.e., k in $H_D[k]$) will we get the minimum magnitude of gain from this controller? Similarly (still independent of window size), what is the maximum magnitude of gain this controller provides, and what is the period (in samples/cycle) of the sinusoid it provides that gain to?



* Minimum value for $H_D[k] = 0$, and this happens when $k = 0$ since

$H_d[0] = \sum h[n] = \frac{1}{T_s} - \frac{1}{T_s} = 0$

* Maximum value happens when

$e^{-j\frac{2\pi}{N}\cdot kn} = [1 \;\; -1]$ for $n = 0, 1$

In this case the gain is $\frac{2}{T_s}$, and the period is 2 samples/cycle.

e) **(BONUS: 4 pts)** Given your answer to part d), what types of errors do derivative controllers help to correct?

* Derivative controllers have higher gain at high frequencies, so they will generally correct transient (as opposed to steady-state) errors.