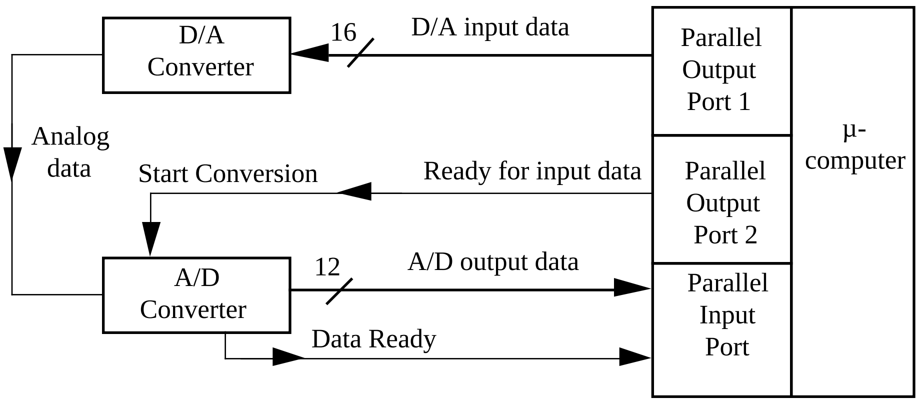
Solutions for Midterm #2 - EECS 145M Spring 1999

1a The following are essential:

- Connect all 16 lines of one parallel output port to the input of the D/A converter
- Connect the analog output of the D/A to the analog input of the A/D
- Connect 12 lines of the output of the A/D to the parallel input port
- Connect 1 line of the other parallel output port to the A/D start conversion input
- Connect the data ready output of the A/D to one of the unused lines of the parallel input port.
- Start A/D conversion under computer program control
- Use the "Data ready" A/D output to signal the program that new data are available



1b

1 Set "Ready for input data" low, which makes "Data Ready" low
2 Start loop over all values of n from 0 to $2^{16} - 1$
3 Write n to the D/A converter
4 Write a low, then a high to "Ready for input data" to start A/D conversion
5 Read "Data ready" in a loop until it goes high
6 When the A/D converter finishes, it strobes the data onto the input port and sets the "Data Ready" line high
7 The program detects this and reads the input port
8 The program sets "Ready for input data" low, which causes the A/D converter to set "Data Ready" low
9 If the A/D output has changed from the last value read (say from m–1 to m), store the value of n, which corresponds to the $V_{m-1,\,m}$ transition voltage.
10 Loop back to step 2
11 Tabulate the difference between the measured transition voltages $V_{m-1,\,m}$ and the ideal transition voltages V(m–1,m) = (m–0.5)(4.095V/4095) = 0.001 V (m–0.5). The maximum value is the maximum absolute accuracy error.

Essential steps: (1) vary all 16 D/A bits; (2) read A/D only after Data Ready has gone high; (3) tabulate D/A input where A/D output changes; (4) compare transition voltages with ideal

[3 points off if only 12 D/A bits are varied. The transition voltages (or the center of the steps) cannot be determined accurately unless more than 12 bits of the accuracy of the 16-bit D/A is used.]

[4 points off if the method is not automatic]

[3 points off if the transition voltages are not measured]

[3 points off if handshaking steps not indicated]

1c As part **1b** above, but compare the measured transition voltages $V_{m-1,\,m}$ (as a function of m) with the straight line passing between the first measured transition voltage $V_{0,1}$ and the last measured transition voltage $V_{4094,\,4095}$. The largest deviation is the maximum linearity error.

[3 points off if the straight line is defined in terms of V_{ref}^{-} and V_{ref}^{+} (which is required for the absolute accuracy error). The maximum linearity error requires a straight line that passed through the *measured* end points]

1d As part **1b** above, but compare the A/D step sizes $V_{m,\,m-1} - V_{m-1,\,m}$ with their average value. The largest deviation is the maximum differential linearity error. Alternatively, the A/D step sizes could be determined as the number of successive D/A inputs that produce the same A/D output.

Note: It was essential to use the concept of a "table of transition voltages" to answer parts **b** and **c** of this problem.

1e Since the D/A has an absolute accuracy of ±1LSB, and its step size is 16 times finer than the average step size of the A/D, this design can measure the A/D transition voltages to an accuracy of ±1/16 of the A/D LSB. Therefore, the accuracy is ±1/32 A/D LSB for the maximum absolute and linear errors and ±1.414/32 A/D LSB for the maximum differential linearity error (difference between two random errors).

Note: Due to a typo on the 1997 midterm #2 solutions, ±1 was also accepted

2a Filter gain >0.99 for frequencies <78,400 Hz

[1 point off for giving a single frequency rather than a range]

2b Filter gain <0.01 for frequencies >177,800 Hz

[1 point off for giving a single frequency rather than a range]

2c $S = M\,\Delta t = M/f_s = 2^{16}/2^{18}$ Hz = 0.25 s

2d H_0 corresponds to 0 Hz (d.c.); H_1 corresponds to 1/S = 4 Hz

2e The FFT produces coefficients H_n, where n = 0 to M–1. Therefore, the coefficient with the highest index is H_{M-1} or $H_{65,535}$, which corresponds to 4 Hz.

[2 points off for H_M and 0 Hz] [3 points off for H_M and 2^{18} Hz]

2f The FFT coefficient that corresponds to the highest frequency is $H_{M/2}$ or $H_{32,768}$. The corresponding frequency is (M/2)/S = 131,072 Hz

2g For a 4,000 Hz sinewave, the primary FFT coefficients are H_{1000} and H_{M-1000}. Additional neighboring coefficients H_{999}, H_{1001}, H_{M-999}, and H_{M-1001} are non-zero (actually half the value of the primary coefficients) due to the side lobes produced by the Hanning window.

[1 point off for omitting side lobes] [4 points off for omitting harmonics]

2h For a 4,000 Hz symmetric square wave, a sequence of harmonics will appear at odd multiples of the 4,000 Hz fundamental. So H_{k1000} and $H_{M-k1000}$ would be non-zero, and the Hanning side lobes would be at $H_{k1000-1}$, $H_{k1000+1}$, $H_{M-k1000-1}$, and $H_{M-k1000+1}$.

[1 point off for omitting side lobes] [4 points off for omitting harmonics]

2i For a 4,002 Hz sinewave, H_{1000}, H_{1001}, H_{M-1000}, and H_{M-1001} would be non-zero and of equal magnitude, and the Hanning side lobes would appear at H_{999}, H_{1002}, H_{M-999} and H_{M-1001}.

[2 points off for omitting side lobes] [2 points off for omitting H_{M-1000} and H_{M-1001}]

2j The primary 4,000 Hz sinewave would produce non-zero values at H_{999}, H_{1000}, and H_{1001}. A second smaller sinewave of slightly higher frequency 4,000 + 4m Hz would produce non-zero values at $H_{1000+m-1}$, H_{1000+m}, and $H_{1000+m+1}$ (there are also complex conjugate coefficients at H_{M-1000}, etc.). For the smaller sinewave to appear as a separate peak, the coefficient H_{1001} must be below the coefficient at $H_{1000-m-1}$, which requires 1001 < 1000 +m –1, or m >2. The smallest value of m we can have is 3, which corresponds to a frequency 12 Hz above 4,000 Hz.

[4 points off for 4 Hz] [3 points off for 8 Hz] [both 12 Hz and 16 Hz were accepted]

2k A sinewave of frequency M – 84,000 Hz ($M = 2^{18}$) = 178,144 Hz will produce non-zero coefficients at H_{20999}, H_{21000}, H_{21001}, $H_{M-20999}$, $H_{M-21000}$, and $H_{M-21001}$. A sinewave of frequency 84,000 Hz will produce non-zero coefficients at exactly the same frequency indexes. This is an example of how a higher frequency can alias to a lower frequency. However, the 84,000 Hz sinewave will be only slightly reduced by the anti-aliasing filter (gain >0.90, while the 178,144 Hz sinewave will be greatly reduced (gain ≈0.01). So the coefficients will be about 100 times smaller for the 178,144 Hz sinewave.

[3 points off for realizing that both frequencies produce the same non-zero magnitudes but stating that the magnitudes are the same]

[3 points off for giving a magnitude ratio of 100 but not giving the non-zero coefficients]

Note: a common mistake was to divide 178,144 Hz by 4 to get the frequency index- this is wrong because all frequencies above the Nyquist limit of 2^{17} Hz = 131,072 Hz are aliased to lower frequencies.

Another common mistake was that 178,144 Hz aliases to 178,144 Hz – 131,072 Hz = 47,072 Hz. Actually H_{M-m} aliases to H_m so 178,144 Hz aliases to 84,000 Hz.

2l To reduce the answer to 2j by a factor of two (i.e. to 6 Hz), sample for twice as long.

To reduce the answer to 2k by a factor of two (i.e. to 200 times smaller), increase the number of stages in the anti-aliasing filter.

[Both answers were accepted]

Midterm #2 class statistics:

Problem	max	average	rms
1	50	43.7	7.2
2	50	31.6	6.9
total	100	75.4	11.5

Grade distribution:

Range	number	*approximate* letter grade
31-40	0	F
41-50	1	F
51-60	1	D
61-70	3	C
71-80	8	B
81-90	9	A
91-100	1	A+