## UNIVERSITY OF CALIFORNIA

College of Engineering
Electrical Engineering and Computer Sciences
Berkeley


## Spring 1997 FINAL EXAM (May 23)

Answer the questions on the following pages completely, but as concisely as possible. The exam is to be taken closed book. Use the reverse side of the exam sheets if you need more space. Calculators are OK but not needed. In answering the problems, you are not limited by the particular equipment you used in the laboratory exercises. Many formulae from the course have been provided for you on the last pages.
Partial credit can only be given if you show your work.
FINAL EXAM GRADE :
$\qquad$ (30 max)
2 $\qquad$ (30 max)

3 $\qquad$ (30 max)

4 $\qquad$ (60 max) $\qquad$ (50 max)

TOTAL $\qquad$ (200 max)

Initials $\qquad$
Problem 1 (total 30 points):
Briefly describe the essential differences between the following pairs of terms:
1a. (10 points) Transparent latch vs. sample and hold amplifier

1b. (10 points) Differential linearity error vs. relative accuracy error (of a D/A converter)

1c. (10 points) Frequency aliasing vs. spectral leakage.

Initials $\qquad$

Problem 2 (30 points)
In this course we studied several types of $A / D$ converters:
Tracking (TR) Successive Approximation (SA) Dual Slope (DS)
Flash (FL) Half-flash (HF)
Sigma-delta (SD)

## For each part below, list the correct two letter codes:

2a. (6 points) Which would provide conversion with low relative accuracy error that does not depend on resistor accuracy?

2b. (6 points) Which use an internal D/A converter?

2c. (6 points) Which would provide the highest conversion rate for 8 -bit accuracy?

2d. (6 points) Which would provide the highest conversion rate for 16-bit accuracy?

2e. (6 points) Which require a sample and hold amplifier for maximum sampling rates?
$\qquad$

Problem 3 (total 30 points):
You have a working real-time sampling and Fourier analysis system that uses a Butterworth antialiasing filter.

## Consider the following design changes for improving your system:

1. Increasing the sampling frequency with faster hardware
2. Adding more poles to the low-pass anti-aliasing filter
3. Increasing the corner frequency of the low-pass anti-aliasing filter
4. Adding a Hanning window.
5. Sampling for a longer time
6. Using an $\mathrm{A} / \mathrm{D}$ converter with higher accuracy (more bits)

3a. (10 points) To increase the highest frequency that can be accurately sampled, what design change(s) would be useful? Give a brief reason for each.

3b. (10 points) To reduce the frequency spacing $\Delta f$ between Fourier transform coefficients, what design change(s) would be useful? Give a brief reason for each.

3c. (10 points) For waveforms whose highest frequency components are adequately sampled in frequency but have infinite extent in time, which design change(s) would improve the observation of weak harmonic components whose frequencies are close to a strong harmonic component? Give a brief reason for each.

Initials $\qquad$
Problem 4 (60 points)
Design in quantitative detail a microcomputer-based data acquisition and Fourier transform system that has the following requirements:

Amplitude accuracy of $\pm 0.1 \%$ from 0 Hz to 18 kHz
All higher frequencies contribute less than $0.1 \%$ of their amplitude to the 0 Hz to 18 kHz band.
Low spectral leakage for all input waveforms.
Ability to individually resolve two components of equal amplitude 1 Hz apart
You are provided with the following components:
Microcomputer with disk drive, keyboard, display screen
Counter/timer circuit that can be set by the computer to produce periodic output pulses
I/O port having 16 input and 16 output lines. Reading or writing takes $1 \mu \mathrm{~s}$.
An A/D converter of your choosing that has a "start conversion" input line and a "data available" output line.
A 12-stage Butterworth low-pass filter of your design (see the design table at the end of this exam).

4a. (20 points) Draw the block diagram for the microcomputer, $A / D$, and the lines that connect them. Label all essential components, control lines, and data lines. Provide sufficient detail so that a skilled technician (who has not taken the course) could understand and build your design.

Initials $\qquad$
4b. (10 points) Describe the hardware and software steps needed to acquire and digitize a single sample of the input waveform.

4c. (10 points) What is the minimum corner frequency you could use in the Butterworth filter?.

4d. (10 points) What is the minimum sampling frequency that you can use?

4e. (5 points) What is the minimum time over which the samples could be taken?

4f. (10 points) What is the minimum number of samples?
$4 \mathbf{g}$. (5 points) What is the minimum $\mathrm{A} / \mathrm{D}$ resolution needed, in terms of number of bits?

Initials $\qquad$
Problem 5 (total 50 points):
You have a computer with an analog input/output port and a high-power amplifier with a frequency response that you know:
(1) does not change with time
(2) is accurate for frequencies below $f_{\min }$, and
(3) declines for frequencies above $f_{\min }$, but is more complicated than a simple function such as $f^{-n}$.

Your goal is to determine the analog output port waveform $b(t)$ that will make the high-power amplifier produce an arbitrary output waveform $a(t)$.

5a. (25 points) For the case where $a(t)$ has a known period $T$ and has zero frequency components above $f_{\text {max }}$, describe all the necessary steps that you would use to compute the amplifier input amplitudes $b_{i}$ that would produce the desired amplifier output values $a_{i}$. (Hint: use the FFT and the Fourier convolution theorem.) Provide sufficient detail so that a programmer/technician who has not taken the course could carry out your procedure.

Initials $\qquad$

5b. (25 points) Consider the more general case where (1) $a(t)$ is not periodic, (2) $a(t)$ is described by a series of amplitudes $a_{i}$ stored on a high-capacity optical disk and (3) the amplitudes $a_{i}$ are to be produced by the amplifier with a periodic time interval $\Delta t$. Your task is to determine the finite impulse response (FIR) digital filter that will continuously transform the series $a_{i}$ to a new series $b_{i}$ that when sent through the amplifier will produce the original series $a_{i}$. In other words, find the digital filter that will pre-condition the $a_{i}$ amplitudes to compensate for the limited response of the amplifier. (Hint: use the FIR filter as a convolution.) Provide sufficient detail so that a programmer/technician who has not taken the course could carry out your procedure.

Initials

## Equations, some of which you might find useful:

$G(a)=\frac{1}{\sqrt{2 \pi \sigma^{2}}} \exp \left[-\frac{1}{2}\left(\frac{a-\mu}{\sigma}\right)^{2}\right] \quad \mu \approx \bar{a}=\frac{1}{m} \sum_{i=1}^{m} a_{i}$
$\sigma_{a}^{2}=\operatorname{Var}(a)=\left(\frac{1}{m-1}\right) \sum_{i=1}^{m} R_{i}^{2}=\left(\frac{1}{m-1}\right) \sum_{i=1}^{m}\left(a_{i}-\bar{a}\right)^{2} \quad \operatorname{Var}(\bar{a})=\operatorname{Var}(a) / m$
$t=\frac{\Delta}{\sigma_{\Delta}}=\frac{\bar{a}-\bar{b}}{\sqrt{\sigma_{\bar{a}}^{2}+\sigma_{\bar{b}}^{2}}}=\frac{\bar{a}-\bar{b}}{\sqrt{\sigma_{a}^{2} / m_{a}+\sigma_{b}^{2} / m_{b}}} \quad \sigma_{f}^{2}=\left(\frac{\partial f}{\partial a_{1}}\right)^{2} \sigma_{a 1}^{2}+\left(\frac{\partial f}{\partial a_{2}}\right)^{2} \sigma_{a 2}^{2}+\cdots+\left(\frac{\partial f}{\partial a_{n}}\right)^{2} \sigma_{a n}^{2}$
$f=k(a+b) \quad \sigma_{f}^{2}=k^{2}\left(\sigma_{a}^{2}+\sigma_{b}^{2}\right) \quad f=k a b \quad \sigma_{f}^{2} / f^{2}=\sigma_{a}^{2} / a^{2}+\sigma_{b}^{2} / b^{2}$
$R_{i}=a+b n_{i}-V_{i} \quad V_{\mathrm{rms}}=\sqrt{\frac{1}{m} \sum R_{i}^{2}}$
$a=\frac{s t-r q}{m s-r^{2}} \quad$ and $\quad b=\frac{m q-r t}{m s-r^{2}} \quad$ where $\quad r=\sum n_{i} \quad s=\sum n_{i}^{2} \quad q=\sum n_{i} V_{i} \quad t=\sum V_{i}$
$V(n)=V_{\text {ref }}^{-}+n\left(\frac{V_{\text {ref }}^{+}-V_{\text {ref }}^{-}}{2^{N}}\right)=V_{\min }+n\left(\frac{V_{\text {max }}-V_{\text {min }}}{2^{N}-1}\right)$
$n=\left[\frac{V-V_{\text {ref }}^{-}}{\Delta V}+\frac{1}{2}\right]_{\text {INTEGER }} \quad V(n-1, n)=V_{\text {ref }}^{-}+(n-0.5) \Delta V \quad \Delta V=\frac{V_{\text {ref }}^{+}-V_{\text {ref }}^{-}}{2^{N}-1}$
$H(f)=\int_{-\infty}^{\infty} h(t) e^{-j 2 \pi f t} d t \quad e^{j \theta}=\cos \theta+j \sin \theta$
If $h(t)=\left\{\begin{array}{l}A \text { for }|t| \leq T_{0} / 2 \\ 0 \text { for }|t|>T_{0} / 2\end{array}\right.$, then $H(f)=A T_{0} \frac{\sin \left(\pi T_{0} f\right)}{\pi T_{0} f}$
If $h(t)=0$ for $t<0 ; \quad h(t)=A e^{-t / \tau}$ for $t \geq 0$, then $H(f)=A / \sqrt{1+4 \pi^{2} f^{2} \tau^{2}}$
$H_{n}=\sum_{k=0}^{M-1} h_{k} e^{-j 2 \pi n k / M} \quad h_{k}=\sum_{n=0}^{M-1} \frac{H_{n}}{M} e^{+j 2 \pi n k / M} \quad d B=20 \log _{10}$
$F_{n}=\left|H_{n}\right|=\sqrt{\operatorname{Re}\left(H_{n}\right)^{2}+\operatorname{Im}\left(H_{n}\right)^{2}} \quad \tan \phi_{n}=\operatorname{Im}\left(H_{n}\right) / \operatorname{Re}\left(H_{n}\right)$
For $h_{k}=\sum_{i=0}^{M-1} a_{i} \cos (2 \pi i k / M)+b_{i} \sin (2 \pi i k / M) \quad H_{0}=M a_{0} \quad H_{n}=(M / 2)\left(a_{n}-j b_{n}\right)$
$\mathrm{f}_{\text {max }}=\mathrm{f}_{\mathrm{s}} / 2 \quad \Delta \mathrm{t}=1 / \mathrm{f}_{\mathrm{s}} \quad \mathrm{S}=\mathrm{M} \Delta \mathrm{t} \quad \Delta \mathrm{f}=1 / \mathrm{S} \quad \mathrm{h}(\mathrm{t})=0.5[1.0-\cos (2 \pi \mathrm{t} / \mathrm{S})]$
$y_{i}=A_{1} x_{i-1}+A_{2} x_{i-2}+\ldots+A_{M} x_{i-M}+B_{1} y_{i-1}+\ldots+B_{N} y_{i-N}$
If $a(t)=\int_{-\infty}^{+\infty} b\left(t^{\prime}\right) c\left(t-t^{\prime}\right) d t^{\prime}=b(t) * c(t)$, then FFT(a) $=$ FFT(b) multiplied by FFT(c)

Initials $\qquad$
$f_{\max }=\frac{1}{2^{N+1} \pi T} \quad V(t)=V(0) e^{-t / R C}$
$\left|\frac{V_{\text {out }}}{V_{\text {in }}}\right|=\frac{1}{\sqrt{1+\left(f / f_{c}\right)^{2 n}}}$ (see table below)

|  | 0.999 | 0.99 | 0.9 | 0.707 | 0.01 | 0.001 | 0.0001 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $f / f \mathrm{cc}(n=6)$ | 0.596 | 0.723 | 0.886 | 1.000 | 2.154 | 3.162 | 4.642 |
| $f / f \mathrm{c}(n=8)$ | 0.678 | 0.784 | 0.913 | 1.000 | 1.778 | 2.371 | 3.162 |
| $f / f \mathrm{c}(n=10)$ | 0.733 | 0.823 | 0.930 | 1.000 | 1.585 | 1.995 | 2.512 |
| flfc $(n=12)$ | 0.772 | 0.850 | 0.941 | 1.000 | 1.468 | 1.778 | 2.154 |


| $N=$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2^{N}=$ | 256 | 512 | 1,024 | 2,048 | 4,096 | 8,192 | 16,384 | 32,768 |
|  |  |  |  |  |  |  |  |  |
| $N=$ | 16 | 17 | 18 | 19 |  |  |  |  |
| $2^{N}=$ | 65,536 | 131,072 | 262,144 | 524,288 |  |  |  |  |

## Have a pleasant summer!



