Solutions for Midterm #2 - EECS 145M Spring 2006

1.1 Filter gain >0.99 for frequencies <78,400 Hz

1.2 Filter gain <0.01 for frequencies >177,800 Hz

1.3 $S = M\,\Delta t = M/f_s = 2^{16}/2^{18}$ Hz = 0.25 s

1.4 H_0 corresponds to 0 Hz (d.c.); H_1 corresponds to 1/S = 4 Hz

1.5 The FFT produces coefficients H_n, where n = 0 to M–1. Therefore, the coefficient with the highest index is H_{M-1} or $H_{65,535}$, which corresponds to 4 Hz.

[2 points off for H_M and 0 Hz] [3 points off for H_M and 2^{18} Hz]

1.6 The FFT coefficient that corresponds to the highest frequency is $H_{M/2}$ or $H_{32,768}$. The corresponding frequency is (M/2)/S = 131,072 Hz

1.7 For a 4,000 Hz sinewave, the primary FFT coefficients are H_{1000} and H_{M-1000}. Additional neighboring coefficients H_{999}, H_{1001}, H_{M-999}, and H_{M-1001} are non-zero (actually half the value of the primary coefficients) due to the side lobes produced by the Hann window.

[2 points off for omitting side lobes] [2 points off for omitting H_{M-999}, H_{M-1000}, and H_{M-1001}]

1.8 For a 4,000 Hz symmetric square wave, a sequence of harmonics will appear at odd multiples of the 4,000 Hz fundamental. So H_{k1000} and $H_{M-k1000}$ would be non-zero, and the Hann side lobes would be at $H_{k1000-1}$, $H_{k1000+1}$, $H_{M-k1000-1}$, and $H_{M-k1000+1}$.

[1 point off for omitting side lobes] [4 points off for omitting harmonics]

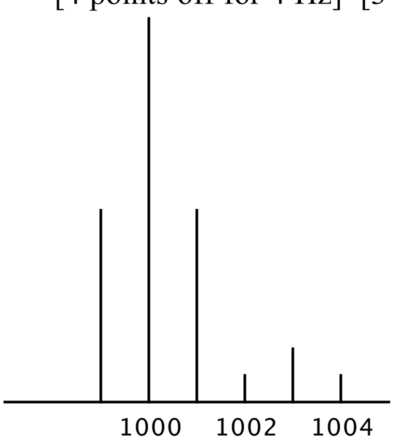
1.9 For a 4,002 Hz sinewave, H_{1000}, H_{1001}, H_{M-1000}, and H_{M-1001} would be non-zero and of equal magnitude, and the Hann side lobes would appear at H_{999}, H_{1002}, H_{M-999} and H_{M-1001}.

[2 points off for omitting side lobes] [2 points off for omitting H_{M-1000} and H_{M-1001}]

[4 points off for stating that all coefficients are non-zero]

1.10 The primary 4,000 Hz sinewave would produce non-zero values at H_{999}, H_{1000}, and H_{1001}. A second smaller sinewave of slightly higher frequency 4,000 + 4m Hz would produce non-zero values at $H_{1000+m-1}$, H_{1000+m}, and $H_{1000+m+1}$ (there are also complex conjugate coefficients at H_{M-1000}, etc.). For the smaller sinewave to appear as a separate peak, there must be a valley between the coefficient H_{1001} and the coefficient at H_{1000+m}, which requires 1000 + m > 1002, or m >2. The smallest value of m we can have is 3, which corresponds to a frequency 12 Hz above 4,000 Hz.

[4 points off for 4 Hz] [3 points off for 8 Hz] [both 12 Hz and 16 Hz were accepted]



1.11 A sinewave of frequency 4M – 84,000 Hz = 178,144 Hz will produce non-zero coefficients at H_{20999}, H_{21000}, H_{21001}, $H_{M-20999}$, $H_{M-21000}$, and $H_{M-21001}$.

$M = 2^{16} = 65,536$. $M - 21,000 = 44,536$.

A sinewave of frequency 84,000 Hz will produce non-zero coefficients at exactly the same frequency indexes. This is an example of how a higher frequency can alias to a lower frequency. However, the 84,000 Hz sinewave will be only slightly reduced by the anti-aliasing filter (gain >0.90, while the 178,144 Hz sinewave will be greatly reduced (gain ≈0.01). So the coefficients will be about 100 times smaller for the 178,144 Hz sinewave.

[3 points off for not stating the non-zero coefficients]

[1 point off for omitting H_{20999}, H_{21001}, $H_{M-20999}$, and $H_{M-21001}$]

[3 points off for stating that the magnitudes are the same for sampling 178,144 and 84,000 Hz]

1.12 To reduce the answer to 1.10 by a factor of two (i.e. to 6 Hz), sample for twice as long.

[2 points off for doubling the sampling frequency, which increases the number of Fourier coefficients but not the frequency spacing $\Delta f = 1/S$]

[2 points off for reducing the sampling frequency]

2.1 v = 3 m/s f = 100 kHz (1 + 3/300) = 101,000 Hz

v = 30 m/s f = 100 kHz (1 + 30/300) = 110,000 Hz

v = 60 m/s f = 100 kHz (1 + 60/300) = 120,000 Hz

The exact calculation gives 101,010; 111,111; 111,235; and 125,000 Hz; both were OK.

2.2 $\Delta f = 100$ Hz, so the minimum length of the sampling window must be $S = 1/\Delta f = 0.01$ s

2.3 Sample for a whole number of 100 kHz cycles. Since a windowing function was not used, this completely eliminates spectral leakage for the 100 kHz primary signal. The much smaller echo signal would still have spectral leakage, but the peak would not be obscured.

[2 points off for increasing the sampling window S, this helps somewhat, but for a non-integer number of 100 kHz cycles, the spectral leakage will be severe because the primary has 100 times the amplitude of the echo signal]

[4 points off for using an anti-aliasing filter since aliasing is only a problem for the white noise]

[4 points off for reducing the sampling frequency, which does not reduce spectral leakage]

[4 points off for using the Hann window, which was specifically excluded in the problem statement]

2.4 For n = 8 and $G_1 = 0.9$, $f_1/f_c = 0.913$. At 60 m/s $f_1 = 120$ kHz, and $f_c = 135.4$ kHz

2.5 For n = 8 and $G_2 = 0.01$, $f_2/f_c = 1.778$, and $f_2 = 233.7$ kHz

The minimum sampling frequency $f_s = f_1 + f_2 = 353.7$ kHz

2.6 The minimum number of samples is the product of

the minimum sampling window (S = 0.01 s from part 2.2) and

the minimum sampling frequency ($f_s = 353.7$ kHz from part 2.5) = 3537 samples.

The next power of two is 4096 samples.

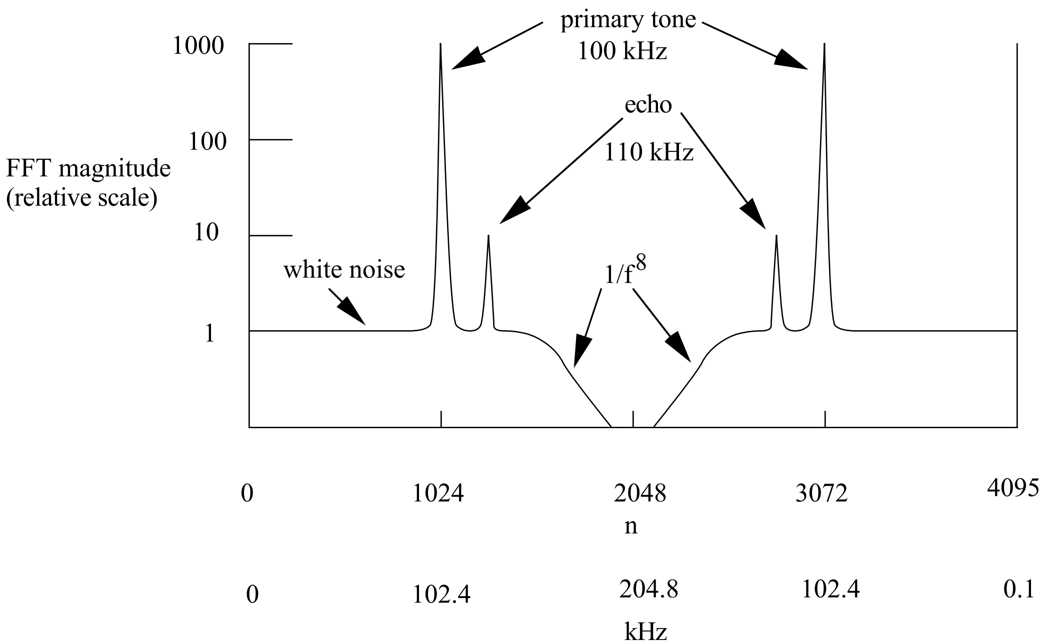
2.7 The echo is 100 times smaller than the 100 kHz tone and the white noise is 10 times smaller than the peak echo. The low pass filter falls off as $1/f^8$.

[2 points off if no FFT frequency index] [2 points off if no frequency scale in Hz]

[2 points off if white noise not shown] [4 points off if primary tone or echo not shown]

[3 points off for blank vertical scale] [2 points if vertical scale is not numbered]

[2 points off if effect of the low-pass filter not shown]



This exam showed that many students are not clear on

(1) the difference between aliasing and spectral leakage

(2) that the difference between the frequencies of neighboring Fourier coefficients Δf is equal to 1/S and not related to the sampling frequency

EECS145M Midterm #2 class statistics:

Problem	max	average	rms
1	54	45.1	8.0
2	46	40.0	7.2
total	100	85.1	14.0

Grade distribution:

Range	number	*approximate* letter grade
45-49	1	D
50-54	0	D+
55-59	0	C-
60-64	0	C
65-69	0	C+
70-74	0	B-
75-79	1	B
80-84	1	B+
85-89	3	A-
90-94	3	A
95-99	2	A+
100	0	A+