Midterm #1 Solutions – EECS 145L Fall 2010

1.1

$V_0 = (k/f)(V_1 - V_2)$ $\qquad$ $V_2 = V_0 R_1/(R_1 + R_2)$

$V_0 = (k/f)V_1 - (k/f)V_0 R_1/(R_1 + R_2)$

$V_0\left[1 + (k/f)R_1/(R_1 + R_2)\right] = V_1(k/f)$

$$G = V_0/V_1 = \frac{k/f}{1 + (k/f)R_1/(R_1 + R_2)} = \frac{1}{(f/k) + R_1/(R_1 + R_2)} = \frac{R_1 + R_2}{(f/k)(R_1 + R_2) + R_1}$$

[10 points off for $G = (R_1 + R_2)/R_1$]

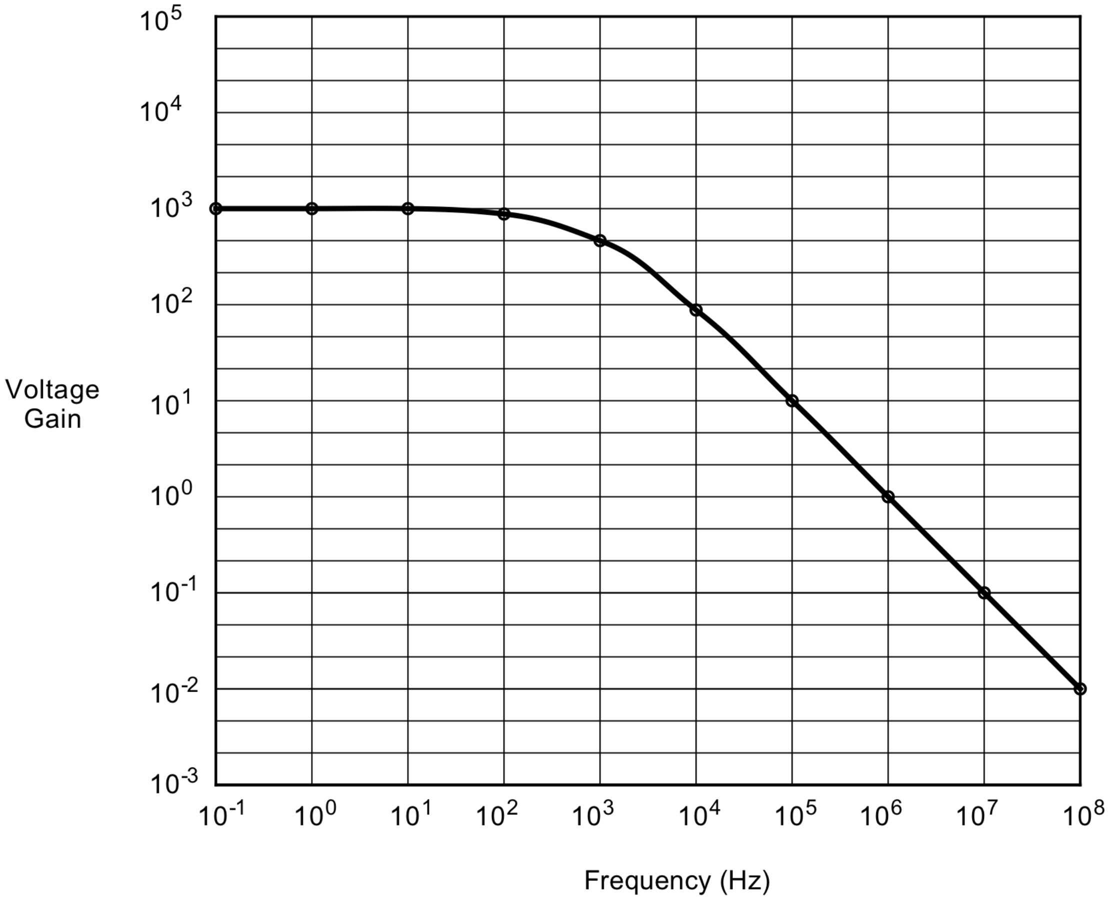
1.2

$$G = V_0/V_1 = \frac{1+999}{1+(1+999)(f/10^6)} = \frac{1000}{1+f/10^3} = \frac{10^6}{10^3+f}$$

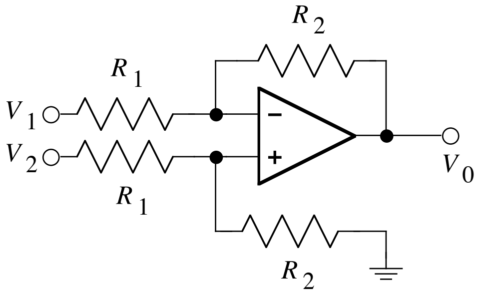
$G = 1000$ at $f << 10^2$ Hz $\qquad$ $G = 909$ at $f = 10^2$ Hz $\qquad$ $G = 500$ at $f = 10^3$ Hz

$G = 90.9$ at $f = 10^4$ Hz $\qquad$ $G = 9.90$ at $f = 10^5$ Hz $\qquad$ $G = 0.999$ at $f = 10^6$ Hz

$G = 0.100$ at $f = 10^7$ Hz $\qquad$ $G = 0.010$ at $f = 10^8$ Hz



2.1



Infinite open-loop op-amp gain: virtual short rule: $V_+ = V_-$

$$\frac{V_1 - V_-}{R_1} = \frac{V_- - V_0}{R_2} \qquad \frac{V_2 - V_+}{R_1} = \frac{V_+}{R_2}$$

$V_1R_2 - V_-R_2 = V_-\mathrm{R}_1 - V_0R_1 \quad V_2R_2 - V_+R_2 = V_+R_1$
Subtracting, $(V_2 - V_1)\, R_2 = V_0R_1$

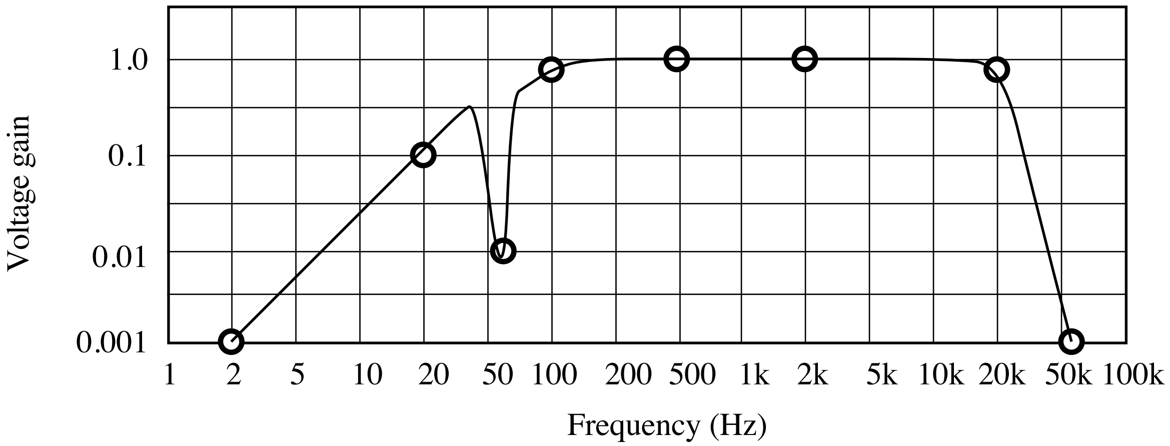
$\boxed{V_0 = (V_2 - V_1)(R_2/R_1)}$

[7 points off if not in terms of resistors]

2.2 Differential gain $V_0 = G_\pm(V_2 - V_1) + G_C(V_2 + V_1)/2$

$\boxed{G_\pm = R_2/R_1 \text{ Since } \mathrm{V}_0 \text{ does not depend on } (\mathrm{V}_1 + \mathrm{V}_2),\ \mathrm{G}_\mathrm{C} = 0}$

3.1



3.2

The LPF needs to have a gain $G_1 = 0.9$ at $f_1 = 20$ kHz and drop to a gain $G_2 = 0.001$ at $f_2 = 52$ kHz. So we need a filter that has $f_2/f_1 < 2.6$.

n	f_1/f_c	f_2/f_c	ratio	
4	0.834	5.623	6.74	n too low
6	0.886	3.162	3.57	n too low

8	0.913	2.371	2.55	n = 8 OK
10	0.930	1.995	2.15	n high, but OK

(20 kHz/0.913) < f_c < (60 kHz/2.371)
21.91 kHz < f_c < 21.93 kHz

$\boxed{\text{LPF } n = 8,\ f_c = 21.92\text{ kHz}}$

[3 points off for f_c = 20 kHz, which would make the gain 0.707 (too low) at 20 kHz]
[3 points off for n = 12 or 14]

The HPF needs to have a gain $G_1 = 0.9$ at 100 Hz and drop to a gain $G_2 = 0.001$ at 2 Hz. So we need a filter that has $f_1/f_2 < 50$

n	f_1/f_c	f_2/f_c	ratio	
2	1.437	0.032	44.9	n = 2 OK
4	1.199	0.178	6.74	n = 4 high, but OK

(2 Hz/0.032) < f_c < (100 Hz/1.437)
62.5 Hz < f_c < 69.6 Hz

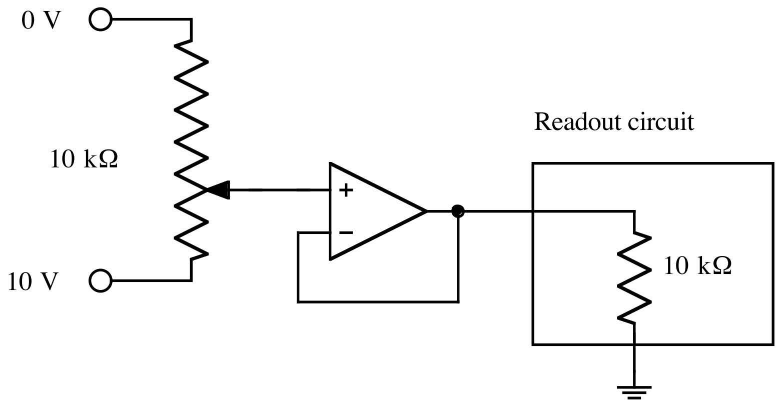
$\boxed{\text{HPF } n = 2,\ f_c = 65\text{ Hz}}$

[3 points off for f_c = 100 Hz, which would make the gain 0.707 (too low) at 100 Hz]

This HPF has a gain just a bit below 0.7 at 60 Hz and does not meet the gain requirement of 0.01. A notch filter with accurate components should provide the necessary low gain.

Note: an alternative solution to the notch filter was to use a 10th or 12th order HPF to reduce the gain from 0.9 at 100 Hz to 0.01 at 60 Hz- although this solution uses 2 or 3 more op-amps, costs more, and has more components that can fail, it was accepted.

4.1



[2 points off for not producing an output that varied from 0 to 10 V as the liquid level varied from 0 m to 10 m]

[5 points off for not providing a buffer amplifier between the 10 kΩ sensor resistor and the readout circuit; this is an inferior design where the output voltage is not linearly proportional to liquid level]

4.2 The relationship between liquid height h (in meters) and output voltage V (in volts) is V = h

An rms uncertainly of 1 mV in V produces an rms uncertainty in liquid height of 1 mm.
[2 points off for mV]

4.3 Determining the change in the liquid level per minute requires making two measurements one minute apart and taking the difference. Making two measurements *a* and *b* exactly one minute apart results in a measurement of the change in liquid level $f = a - b$ (in mm per minute).

$\sigma_a = \sigma_a =$ 1 mm (from part 4.2)

$\sigma_f^2 = \sigma_a^2 + \sigma_b^2 =$ 2 (mm/min)^2

$\boxed{\sigma_f = 1.414 \text{ mm / min}}$ (1.4 mm was accepted for full credit)

[2 points off for 1.414 mV/min]
[3 points off for 1 mm/min or 2 mm/min]
[4 points off for 1 mV/min or 2 mV/min]
[5 points off for 0.01%/min]
[6 points off for 1 mV or 2 mV]

Note 1: The equation sheet said that if $f = k(a - b)$ then $\sigma_f^2 = k^2(\sigma_a^2 + \sigma_b^2)$

Note 2: The rate of change in liquid level is measured in mm/min, not mV or mV/min.

145L midterm #1 grade distribution:

Problem	
1	21.9 (5.1 rms) (25 max)
2	22.2 (6.0 rms) (25 max)
3	26.7 (7.6 rms) (30 max)
4	14.2 (5.6 rms) (20 max)

maximum score = 100
average score = 85.0 (B) (19.2 rms)

< 55	3	F
55-59	0	F
60-64	1	D
65-69	0	D
70-74	0	C
75-79	2	C
80-84	1	B
85-89	3	B
90-94	5	A
95-99	3	A
100	6	A+

EECS average 88.4 (12.1 rms)
BioEng average 81.7 (25.8 rms)