## Midterm \#1 Solutions - EECS 145L Fall 2008

1.1 Electronic Sensor: Device that converts a physical signal into an electrical signal
[3 points off for electronic input only]
[2 points off for electronic or physical input]
1.2 Sensitivity of an electronic sensor: Change in output per unit change in physical quantity being sensed
[2 points off for ability to detect small signals or small changes]
Note: For a physical signal $f$ and an electrical output $V$ the sensitivity is $d V / d f$, a property of the sensor. The smallest detectable signal change is $\Delta \mathrm{f}=\Delta \mathrm{V} /(\mathrm{dV} / \mathrm{df})$ and depends on both the sensitivity and the electronic noise $\Delta \mathrm{V}$.
1.3 Instrumentation Amplifier: Amplifier circuit that has (1) output voltage proportional to the voltage difference between the two inputs , (2) very high input impedance, (3) low output impedance, and (4) constant gain over a large bandwidth
[ 2 points off for omitting constant gain over a frequency range; high input impedance, low output impedance, and differential gain do not distinguish it from the op-amp]
1.4 Differential gain (of an amplifier with two inputs): Change in output voltage divided by the change in the difference in input voltages
1.5 Common mode gain (of an amplifier with two inputs): The gain for signals present at both inputs
1.6 Johnson noise (of a resistor): Random voltage generated in a resistor due to the thermal agitation of electrons within it.

## 2.1



Infinite open-loop op-amp gain: virtual short rule: $V_{+}=V_{-}$
$\frac{V_{1}-V_{-}}{R_{1}}=\frac{V_{-}-V_{0}}{R_{2}} \quad \frac{V_{2}-V_{+}}{R_{1}}=\frac{V_{+}}{R_{2}}$
$V_{1} R_{2}-V_{-} R_{2}=V_{-} \mathrm{R}_{1}-V_{0} R_{1} \quad V_{2} R_{2}-V_{+} R_{2}=V_{+} R_{1}$
Subtracting, $\left(V_{2}-V_{1}\right) R_{2}=V_{0} R_{1}$
$V_{0}=\left(V_{2}-V_{1}\right)\left(R_{2} / R_{1}\right)$
2.2 Differential gain $V_{0}=G_{ \pm}\left(V_{2}-V_{1}\right)+G_{\mathrm{C}}\left(V_{2}+V_{1}\right) / 2$
$G_{ \pm}=R_{2} / R_{1}$ Since $\mathrm{V}_{0}$ does not depend on $\left(\mathrm{V}_{1}+\mathrm{V}_{2}\right), \mathrm{G}_{\mathrm{C}}=0$
3.1


Frequency (Hz)

## 3.2

The LPF needs to have a gain $G_{1}=0.9$ at $f_{1}=20 \mathrm{kHz}$ and drop to a gain $\mathrm{G}_{2}=0.001$ at $\mathrm{f}_{2}=52$ kHz . So we need a filter that has $\mathrm{f}_{2} / \mathrm{f}_{1}<2.6$.

| n | $\mathrm{f}_{1} / \mathrm{f}_{\mathrm{c}}$ | $\mathrm{f}_{2} / \mathrm{f}_{\mathrm{c}}$ | ratio |  |
| :--- | :--- | :--- | :--- | :--- |
| 4 | 0.834 | 5.623 | 6.74 | n too low |
| 6 | 0.886 | 3.162 | 3.57 | n too low |
| 8 | 0.913 | 2.371 | 2.55 | $\mathrm{n}=8$ OK |
| 10 | 0.930 | 1.995 | 2.15 | n high, but OK |

$(20 \mathrm{kHz} / 0.913)<\mathrm{f}_{\mathrm{c}}<(60 \mathrm{kHz} / 2.371)$
$21.91 \mathrm{kHz}<\mathrm{f}_{\mathrm{c}}<21.93 \mathrm{kHz}$
LPF $\mathrm{n}=8, \mathrm{f}_{\mathrm{c}}=21.92 \mathrm{kHz}$
[ 3 points off for $f_{c}=20 \mathrm{kHz}$, which would make the gain 0.707 (too low) at 20 kHz ]
[3 points off for $n=12$ or 14 ]

The HPF needs to have a gain $G_{1}=0.9$ at 100 Hz and drop to a gain $\mathrm{G}_{2}=0.001$ at 2 Hz . So we need a filter that has $\mathrm{f}_{1} / \mathrm{f}_{2}<50$

| n | $\mathrm{f}_{1} / \mathrm{f}_{\mathrm{c}}$ | $\mathrm{f}_{2} / \mathrm{f}_{\mathrm{c}}$ | ratio |  |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 1.437 | 0.032 | 44.9 | $\mathrm{n}=2$ OK |
| 4 | 1.199 | 0.178 | 6.74 | $\mathrm{n}=4$ high, but OK |

$(2 \mathrm{~Hz} / 0.032)<\mathrm{f}_{\mathrm{c}}<(100 \mathrm{~Hz} / 1.437)$
$62.5 \mathrm{~Hz}<\mathrm{f}_{\mathrm{c}}<69.6 \mathrm{~Hz}$
HPF $\mathrm{n}=2, \mathrm{f}_{\mathrm{c}}=65 \mathrm{~Hz}$
[ 3 points off for $\mathrm{f}_{\mathrm{c}}=100 \mathrm{~Hz}$, which would make the gain 0.707 (too low) at 100 Hz ]
This HPF has a gain just a bit below 0.7 at 60 Hz and does not meet the gain requirement of 0.01 . A notch filter with accurate components should provide the necessary low gain.

Note: an alternative solution to the notch filter was to use a 10th or 12th order HPF to reduce the gain from 0.9 at 100 Hz to 0.01 at 60 Hz - although this solution uses 2 or 3 more op-amps, costs more, and has more components that can fail, it was accepted.

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145L midterm \#1 grade distribution:

| Problem |  |
| :--- | :--- |
| 1 | $33.1(2.9 \mathrm{rms})(36 \mathrm{max})$ |
| 2 | $27.4(5.4 \mathrm{rms})(30 \mathrm{max})$ |
| 3 | $32.2(3.5 \mathrm{rms})(34 \mathrm{max})$ |


| maximum score $=$ | 100 |  |
| :--- | ---: | ---: |
| average score $=$ | 92.7 | $(8.8 \mathrm{rms})$ |
|  |  |  |
| $70-74$ | 2 | $\mathrm{C}-$ |
| $75-79$ | 2 | C |
| $80-84$ | 1 | $\mathrm{C}+$ |
| $85-89$ | 3 | $\mathrm{~B}-$ |
| $90-94$ | 4 | B |
| $95-99$ | 8 | $\mathrm{~B}+$ |
| 100 | 10 | A |

Note: The average score among the 14 EECS undergraduates was 95.0. The average score for the 11 BioEng undergraduates was 91.3.

