## Midterm \#1 Solutions - EECS 145L Fall 2004

## 1a

| Johnson Noise | Shot noise |
| :--- | :--- |
| Increases with temperature | Does not depend on temperature |
| Occurs in resistors | Occurs in resistors and conductors |
| Random voltage fluctuations | Current fluctuations caused by |
| caused by thermal agitation of | random fluctuations in the number of <br> electrons per unit time |
| electrons in a resistor |  |

[-2 points for each missing answer out of 4 statements]
[Several students wrote that one was white noise and the other was not. The noise power for both is proportional to $\mathrm{V}_{\mathrm{rms}}{ }^{2}$ or $\mathrm{I}_{\mathrm{rms}}{ }^{2}$ which is proportional to $\Delta \mathrm{f}$. So both are white noise.]

## 1b

| Johnson Noise | Electromagnetic Interference Noise |
| :--- | :--- |
| White (random) noise | Structured noise |
| Due to thermal motion of <br> electrons in a resistor | From other circuits (or lightning) |


| Increases with temperature | Does not depend on temperature |
| :--- | :--- |
| Cannot be cancelled by differential | Can be cancelled by differential <br> amplification |
| Cannot be reduced by conductive <br> shielding | Can be reduced by conductive <br> shielding |

[-2 points for each missing answer out of 4 statements]
[Many students gave two statements that described one difference. Four statements are needed to describe two differences between two items.]

2a Virtual short rule $V_{5}=V_{6}$
$\frac{V_{3}-V_{5}}{R_{3}}=\frac{V_{5}-V_{0}}{R_{4}} \quad \frac{V_{4}-V_{5}}{R_{3}}=\frac{V_{5}}{R_{4}}$
$V_{3} R_{4}-V_{5} R_{4}=V_{5} R_{3}-V_{0} R_{3} \quad V_{4} R_{4}-V_{5} R_{4}=V_{5} R_{3}$
Subtracting, $\left(V_{4}-V_{3}\right) R_{4}=V_{0} R_{3}$
$V_{0}=\left(V_{4}-V_{3}\right)\left(R_{4} / R_{3}\right)$
2b Differential gain $V_{0}=G_{ \pm}\left(V_{4}-V_{3}\right)+G_{C}\left(V_{4}+V_{3}\right) / 2$
$G_{ \pm}=\frac{R_{4}}{R_{3}} \quad$ Since $\mathrm{V}_{0}$ does not depend on $\left(\mathrm{V}_{3}+\mathrm{V}_{4}\right), \mathrm{G}_{\mathrm{C}}=0$
[-2 points for $G_{C} \neq 0$ ]

3a From textbook, example 2.1, page 91:
common mode $\mathrm{V}_{-}=\mathrm{V}_{+}$
Virtual short rule $V_{-}=V_{2}=V_{+}=V_{1}$
No current through R1 and R2: $\mathrm{V}_{-}=\mathrm{V}_{2}=\mathrm{V}_{4}=\mathrm{V}_{+}=\mathrm{V}_{1}=\mathrm{V}_{3}$
$G_{c}=\frac{V_{3}+V_{4}}{V_{-}+V_{+}}=1$
3b From textbook, example 2.2, pages 91 and 92:
Virtual short rule $\mathrm{V}_{-}=\mathrm{V}_{1}$ and $\mathrm{V}_{+}=\mathrm{V}_{2}$
The same current flows through $\mathrm{R}_{1}$ and $\mathrm{R}_{2}: \quad \frac{V_{2}-V_{1}}{R_{1}}=\frac{V_{4}-V_{3}}{R_{1}+2 R_{2}}$
Differential gain of the first stage: $\frac{V_{4}-V_{3}}{V_{+}-V_{-}}=\frac{R_{1}+2 R_{2}}{R_{1}}$
3c From textbook, example 2.2, pages 91 and 92:
$\frac{V_{0}}{V_{+}-V_{-}}=\left(\frac{V_{4}-V_{3}}{V_{+}-V_{-}}\right)\left(\frac{V_{0}}{V_{4}-V_{3}}\right)=\left(\frac{R_{1}+2 R_{2}}{R_{1}}\right)\left(\frac{R_{4}}{R_{3}}\right)$
4a $V_{3}=V_{-}\left(R_{1}+R_{2}\right) / R_{1} \quad V_{4}=V_{+}\left(R_{1}+R_{2}\right) / R_{1}$
$\frac{V_{4}+V_{3}}{V_{+}+V_{-}}=\frac{R_{1}+R_{2}}{R_{1}}$
4b $\frac{V_{4}-V_{3}}{V_{+}-V_{-}}=\frac{R_{1}+R_{2}}{R_{1}}$
4c The new design is inferior because its first stage common mode gain is as high as the differential gain and prone to saturation. The standard design has a first stage common mode gain of only one.

5a

[-2 points for not producing an output that varied from 0 to 10 V as the liquid level varied from 0 m to 10 m ]
[-4 points off for not providing a buffer amplifier between the $10 \mathrm{k} \Omega$ sensor resistor and the readout circuit]
5b Determining the change in the liquid level per minute requires making two measurements one minute apart and taking the difference. The standard deviation of each measurement is 1 mV , which corresponds to 1 mm in liquid level.
$f=(a-b) / \Delta T \quad \sigma_{a}=\sigma_{b}=1 \mathrm{~mm} \quad \sigma_{f}^{2}=\left(\sigma_{a}^{2}+\sigma_{b}^{2}\right) /(\Delta T)^{2}=2(\mathrm{~mm} / \mathrm{min})^{2}$
$\sigma_{f}=1.414 \mathrm{~mm} / \min \quad(1.4 \mathrm{~mm}$ was accepted for full credit)
[-3 points for 1 mV ] [-1 point for 1.4 mV ] [-3 points for 2 mV ]
[-2 points for 1 mm ] [-2 points for 2 mm ]
Note 1: The equation sheet said that if $f=k(a-b)$ then $\sigma_{f}^{2}=k^{2}\left(\sigma_{a}^{2}+\sigma_{b}^{2}\right)$
Note 2: The rate of change in liquid level is measured in $\mathrm{mm} / \mathrm{min}$, not mV .
6

[-1 points for HPF with $n=2$ ] [-2 points for HPF with $n>2$ ]
[-1 points for LPF with $n=3][-2$ points for LPF with $n>3]$

## 145L midterm \#1 grade distribution:

| Problem |  |
| :--- | :--- |
|  |  |
| 1 | $11.6(4.7 \mathrm{rms})(16 \mathrm{max})$ |
| 2 | $19.2(2.0 \mathrm{rms})(20 \max )$ |
| 3 | $15.8(5.5 \mathrm{rms})(20 \max )$ |
| 4 | $15.2(6.1 \mathrm{rms})(20 \max )$ |
| 5 | $8.8(4.8 \mathrm{rms})(16 \max )$ |


| maximum score $=$ | 100 |  |
| :--- | :---: | ---: |
| average score $=$ | 76.8 | $(17.1 \mathrm{rms})$ |
| $30-39$ | 1 | F |
| $40-49$ | 2 | F |
| $50-59$ | 2 | D |
| $60-69$ | 4 | C |
| $70-79$ | 5 | $\mathrm{~B}-$ |
| $80-89$ | 6 | B |
| $90-97$ | 8 | A |

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6
6.2 ( 1.5 rms ) ( 16 max )

98 (max)
1
A+

3 graduate students: average $=79.7$
8 BioEngineering undergraduates: average $=78.6$
13 EECS undergraduates: average $=73.3$

