# EECS 145L Final Examination Solutions (Fall 2004) 

UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering, Electrical Engineering and Computer Sciences Department
1.1 The ideal op-amp amplifier has (1) differential amplification (2) infinite gain at all frequencies (3) infinite input impedance (4) zero output impedance
[each of the four items was worth 2 points] [it was important to specify or show differential amplification because there is also a possible common mode gain, which is zero]
1.2 The full-wave precision rectifier has one input and one output. The output is equal to the absolute value of the input, even for very small ( mV ) input voltages.
[2 points off for not mentioning that it works for inputs $<0.6 \mathrm{~V}$, which distinguishes it from a fullwave rectifier that consists only of diodes; most students missed this one]
[2 points off for describing output for negative input only]
1.3 PID control is an algorithm that generates a control signal as a linear combination of the error signal P , its time integral I , and its time derivative D .
[3 points off for not mentioning the set point or error signal, which are essential for any control system]
[1 point off for not explaining the integral component of the control signal
[2 point off for not explaining the differential component of the control signal]
[2 point off for not explaining the proportional component of the control signal]
1.4 The electromagnetic isolation amplifier modulates the input signal with a high frequency carrier and transmits it to the output stage for demodulation. Transmission uses an air-core transformer that does not pass d.c. or 60 Hz .
1.5 The thermistor is a semiconducting temperature sensor whose resistance decreases with increasing temperature as $\exp (\beta / T)$
[2 points off for not stating which way the resistance changes with temperature]
2 The most straightforward solution was to isolate each 1 kHz and 2 kHz signal by using sharp $\mathrm{n}=10$ LPF and HPF in series. For a LPF with $n=10$, the gain $=0.99$ at $\mathrm{f} / \mathrm{fc}=0.823$. For a HPF with $\mathrm{n}=$ 10 , the gain $=0.99$ at $\mathrm{f} / \mathrm{fc}=1.215$.
The 1 kHz signal would be selectively passed by using a LPF with $\mathrm{fc}=1.22 \mathrm{kHz}$ in series with a HPF with $\mathrm{fc}=0.823 \mathrm{kHz}$.
The 2 kHz signal would be selectively passed by using a LPF with $\mathrm{fc}=2.44 \mathrm{kHz}$ in series with a HPF with $\mathrm{fc}=1.646 \mathrm{kHz}$.
[other high values of n were also accepted]
[8 points off for using LPF and HPF to separate the 1 kHz and 2 kHz signals from the 1 V p-p background but not separating them on separate outputs]
[6 points off for using a LPF to extract the 1 kHz signal and a HPF to extract the 2 kHz signal- this does not sufficiently remove the 1 V p-p background]
[16 points off for using a 1 kHz notch filter to generate signal 1 and a 2 kHz notch filter to generate signal 2- this results in useless signals that contain backgrounds and not the signals of interest]

Another solution was to use $\operatorname{LPF}\left(\mathrm{f}_{\mathrm{c}} \approx 2.5 \mathrm{kHz}\right)$ and $\operatorname{HPF}\left(\mathrm{f}_{\mathrm{c}} \approx 0.8 \mathrm{kHz}\right)$ to separate the 1 kHz to 2 kHz band from the total signal and then to use a 2 kHz notch filter to extract signal 1 and to use a 1
kHz notch filter to extract signal 2. This solution does not remove all of the background in the 1 to 2 kHz band, but was accepted for full credit.
The solution with the best background elimination removed the 1 kHz and 2 kHz signals with notch filters and subtracted the results from the original signal with difference amplifiers, as shown below.

[The delay compensates for the delay through the op-amps in the notch filters- it was not required for full credit]
[16 points off for using notch filters in a way that removes the 1 kHz and 2 kHz signals from the outputs]

3a Differential gain 1000, bandwidth 10 kHz
[ 3 points off for Gain $=10,000$, bandwidth 1 kHz ]
[3 points off for Gain $=100$, bandwidth 100 kHz ]
3b Output $\mathrm{V}_{\mathrm{rms}}=\left(4 \mathrm{nV} \mathrm{Hz}^{-1 / 2}\right)\left(100 \mathrm{~Hz}^{1 / 2}\right)(1000)=0.4 \mathrm{mV}$ in 10 kHz
[2 points off for input noise rather than output noise]
[3 points off for not taking the square root of the bandwidth]
3c We want a Butterworth low-pass filter with a gain of $G_{1}=0.99$ at $f_{1}=1 \mathrm{kHz}$ and $G_{2}=0.01$ at $f_{2}=$ 2 kHz .

| n | $\mathrm{f}_{1} / \mathrm{f}_{\mathrm{c}}$ | $\mathrm{f}_{\mathrm{c} 1}$ | $\mathrm{f}_{2} / \mathrm{f}_{\mathrm{c}}$ | $\mathrm{f}_{\mathrm{c} 2}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 0.723 | 1.383 kHz | 2.154 | 0.929 kHz | $\mathrm{fc} 1>\mathrm{fc} 2 ; \mathrm{n}$ too low |
| 8 | 0.784 | 1.276 kHz | 1.778 | 1.125 kHz | $\mathrm{fc} 1>\mathrm{fc} 2 ; \mathrm{n}$ too low |
| 10 | 0.823 | 1.215 kHz | 1.585 | 1.262 kHz | $\mathrm{fc} 1<\mathrm{fc} 2 ; \mathrm{n}=10 \mathrm{OK}$ |

The order is 10 and a corner frequency between 1.22 kHz and 1.26 kHz is OK . (order 12 also accepted)
[2 points off for order 8]
[ 4 points off if output noise not given or determined by multiplying the answer from 3 b by 0.01 or 0.99]

After amplification and filtering, the output noise would be $\mathrm{V}_{\mathrm{rms}}=(4 \mathrm{nV} \mathrm{Hz}-1 / 2) \operatorname{sqrt}(1.24 \mathrm{kHz})$ $(1000)=\left(4 \mathrm{nV} \mathrm{Hz}^{-1 / 2}\right)\left(35.3 \mathrm{~Hz}^{1 / 2}\right)(1000)=0.141 \mathrm{mV}$ in 10 kHz
So the filtering reduced the output noise from $\pm 0.4 \mathrm{mV}$ to $\pm 0.14 \mathrm{mV}$
3d The best way to reduce the 60 Hz interference from the middle of a band of frequencies of interest is to use a notch filter. The common mode rejection ratio of 60 dB means that the common mode gain

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is $1000 / 1000=1$. So the instrumentation amplifier 60 Hz output will be $\pm 10 \mathrm{mV}$ from a common mode input of $\pm 10 \mathrm{mV}$. The output due to a differential 60 Hz interference is $( \pm 0.01 \mathrm{mV})(1000)=$ $\pm 10 \mathrm{mV}$. In the worst case, these are in phase, producing a total of $\pm 20 \mathrm{mV}$. A notch filter can reduce this total by a factor of typically 30 , to $\pm 0.7 \mathrm{mV}$.
[Any value between 0.1 and 2 mV was accepted for full credit]
[2 points off for including one 10 mV and not the other]
[5 points off if input noise not given or not based on 60 Hz interference]
[5 points off if output noise not given]
[10 points off for using a HPF, which removes the important earthquake frequencies below 60 Hz ]
3e [OK to reverse order of low pass and notch filters]

[4 points off if amplifier not in circuit]

### 4.1 Case $V_{1}<0$

op-amp has positive output so only $\mathrm{D}_{1}$ conducts to close negative feedback loop
$\mathrm{V}_{2}=0$ by virtual ground
$\mathrm{V}_{0}=-\mathrm{V}_{1}$
$\mathrm{V}_{3}=-\mathrm{V}_{1}+0.6 \mathrm{~V}$
[4 points off for $\mathrm{V}_{3}=\mathrm{V}_{1}$ ] [4 points off for $\mathrm{V}_{3}=0.6 \mathrm{~V}$ ] [6 points off for $\mathrm{V}_{0}=0$ ]
[6 points off for $\mathrm{V}_{2} \neq 0$ ] [6 points off for $\left|\mathrm{V}_{3}\right|>\mathrm{V}_{1}$ ] [6 points off for $\mathrm{V}_{3}=0$ ]

### 4.2 Case $V_{1}>0$

op-amp has negative output so only $\mathrm{D}_{2}$ conducts to close negative feedback loop
$V_{2}=0$ by virtual ground
$\mathrm{V}_{0}=0$ due to direct connection to $\mathrm{V}_{2}$
$V_{3}=-0.6 \mathrm{~V}$
[2 points off for $\mathrm{V}_{3}=+0.6 \mathrm{~V}$ ] [4 points off for $\mathrm{V}_{3}=0$ ] [6 points off for $\mathrm{V}_{0}= \pm \mathrm{V}_{1}$ ]
[6 points off for $V_{2} \neq 0$ ] [6 points off if $V_{3}$ depends on $V_{1}$ ]

5a


If the thermistor is $<50 \mathrm{C}, \mathrm{V}_{1}<0, \mathrm{~V}_{2}>0, \mathrm{~V}_{3}=0$
If the thermistor is $>50 \mathrm{C}, \mathrm{V}_{1}>0, \mathrm{~V}_{2}=0, \mathrm{~V}_{3}>0$
The difference amplifier was not needed if the thermistor bridge was set for 0 V output at 50 C . This design makes it more difficult to change the set point, however.

## 5b

- Initially set point is changed to 50 C , error signal $\mathrm{V}_{1}$ is negative and heater is activated
- When temperature gets slightly above 50 C , the error signal $\mathrm{V}_{1}$ becomes positive, and the cooler turns on briefly to bring the temperature below 50C
- As chemical reaction proceeds, tanks heats above 50 C , the error signal $\mathrm{V}_{1}$ becomes positive, and the cooler turns to hold the temperature at 50 C
- After the chemical reaction completes, error signal $\mathrm{V}_{1}$ is negative and heater is activated to maintain 50C


## 145L FINAL EXAM GRADE STATISTICS

| Problem | 1 | 2 | 3 | 4 | 5 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Average | 31.4 | 18.0 | 45.5 | 26.9 | 37.0 | 158.7 |
| rms | 4.8 | 7.7 | 6.7 | 12.4 | 7.6 | 22.5 |
| Maximum | 40 | 24 | 56 | 40 | 40 | 200 |

Total score distribution:

| $100-109$ | 0 | $110-119$ | 1 | $120-129$ | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $130-139$ | 3 | $140-149$ | 4 | $150-159$ | 2 |
| $160-169$ | 5 | $170-179$ | 4 | $180-189$ | 4 |
| $190-199$ | 1 | 200 | 0 |  |  |

## 145L COURSE GRADE STATISTICS

| Grade | Undergraduate <br> Scores | Graduate <br> Scores |
| :--- | :---: | :---: |
| A+ | 944 |  |
| A | $926,933,939$ | 924 |
| A- | $892,892,895,901,906,907,908,916$ | 914 |
| B+ | 872 | 877 |
| B | $830,832,837,839,840,841$ |  |
| B- | 805 |  |
| C+ | $779,781,798$ |  |
| C | 765 |  |
| C- |  |  |
| D+ |  | 1000 |
| D |  | 905.0 |
| D- | 1000 | 24.8 |

Bioengineering undergraduate average $=865.8$, $\mathrm{rms}=32.7$
EECS undergraduate average $=869.7, \mathrm{rms}=64.4$

Note: the average grade for the lab report $4,6,12,14$ series was 87.5 and the average grade for the lab report $5,11,13,15$ series was 89.0 . The difference was less than one standard deviation of the difference. No adjustment was necessary.

