Midterm #1 Solutions – EECS 145L Fall 2000

1a

$V_0 = (k/f)(V_1 - V_2) \qquad V_2 = V_0 R_1/(R_1 + R_2)$

$V_0 = (k/f)V_1 - (k/f)V_0 R_1/(R_1+R_2)$

$V_0\left[1 + (k/f)R_1/(R_1+R_2)\right] = V_1(k/f)$

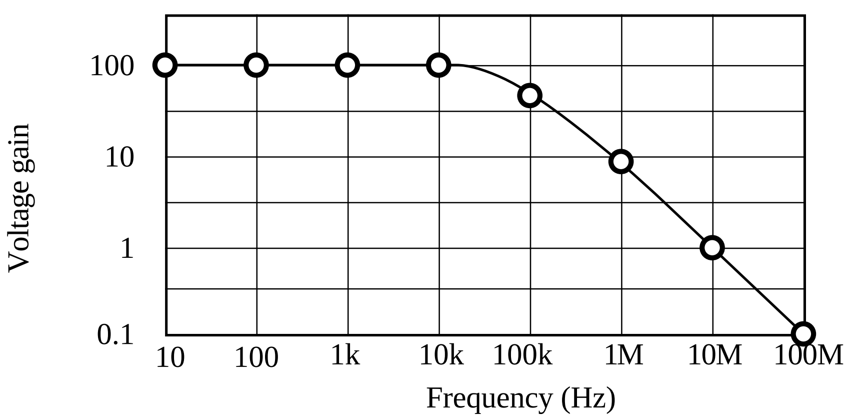
$$G = V_0/V_1 = \frac{k/f}{1+(k/f)R_1/(R_1+R_2)} = \frac{1}{(f/k)+R_1/(R_1+R_2)} = \frac{R_1+R_2}{(f/k)(R_1+R_2)+R_1}$$

[10 points off for $G = (R_1 + R_2)/R_1$]

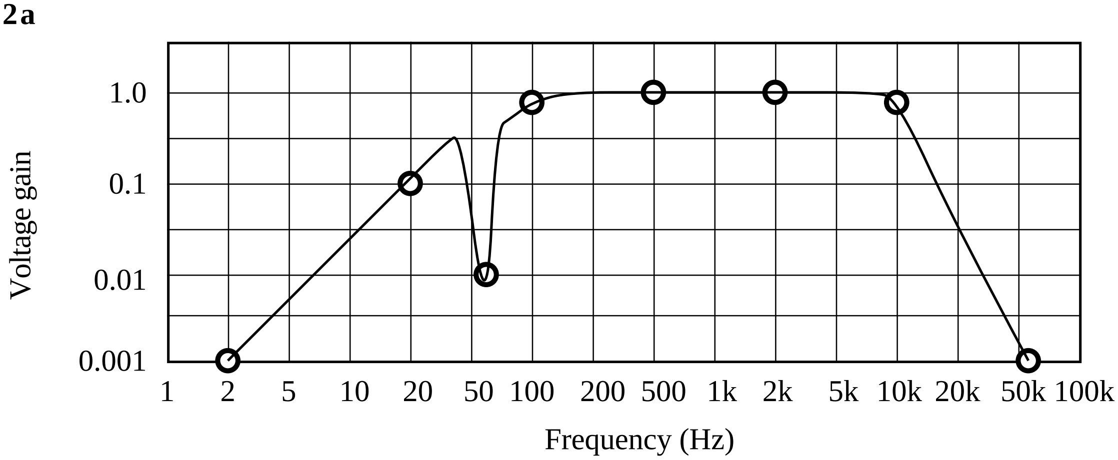
1b

$$\boxed{G = V_0/V_1 = \frac{100}{1+f/10^5\text{Hz}}}$$

$G = 100$ for $f \ll 10^5$ Hz	$G = 91$ for $f = 10^4$ Hz	$G = 50$ for $f = 10^5$ Hz
$G = 9.1$ for $f = 10^6$ Hz	$G = 1$ for $f = 10^7$Hz	$G = 0.1$ for $f = 10^8$ Hz



2a



2b

The LPF needs to have a gain of 0.9 at 20 kHz and drop to a gain of 0.001 at 52 kHz (frequency ratio 2.6). The Butterworth LPF table shows that an **8th order low pass Butterworth filter** has f/f_c = 0.913 at a gain of 0.9 and f/f_c = 2.371 at a gain of 0.001. This is a frequency ratio of 2.6, as required. A smaller order number has a frequency ratio that is too small and a higher order number would be excessive. The corner frequency $\boldsymbol{f_c}$ **= 20 kHz/0.913 = 21.9 kHz**.

[n = 10 or 12 was also accepted]

The HPF needs to have a gain of 0.9 at 100 Hz and drop to a gain of 0.001 at 2 Hz (frequency ratio 50). The Butterworth HPF table shows that a **second order high pass Butterworth filter** has f/f_c = 1.437 at a gain of 0.9 and f/f_c = 0.032 at a gain of 0.001. This is a frequency ratio of 45, and meets the requirement. A HPF of order 1 would not meet the requirement and a higher order filter would be excessive. The corner frequency f_c = 100 Hz/1.437 = 70 Hz.
[n = 4 was also accepted]

The HPF has a gain just a bit below 0.7 at 60 Hz and does not meet the gain requirement of 0.01. A notch filter with accurate components should provide the necessary low gain.

[3 points off for using a 10 or 12 pole HPF rather than a notch filter to reduce the gain from 0.9 at 100 Hz to 0.01 at 60 Hz- this uses 4 or 5 more op-amps and is inefficient and expensive]

3a

To measure common mode gain, connect both inputs of the instrumentation amplifier to a sine wave generator and measure V_{in} and V_{out} vs frequency. $G_c = V_{out}/V_{in}$.

To measure differential gain, ground one input and connect the other to a sine wave generator and measure V_{in} and V_{out} vs frequency. The differential input is V_{in} and the common mode is $V_{in}/2$.
From $V_{out} = G_{\pm}V_{in} + G_c V_{in}/2$ and G_c measured above, compute $G_{\pm}$.
[4 points off for not providing two inputs]
[1 point off for not correcting for the common mode input $V_{in}/2$. Note that the common mode gain couples both the positive and negative inputs equally to the output.]

$$V_0 = G_+V_+ + G_-V_- = G_{\pm}(V_+ - V_-) + G_c(V_+ + V_-)/2$$

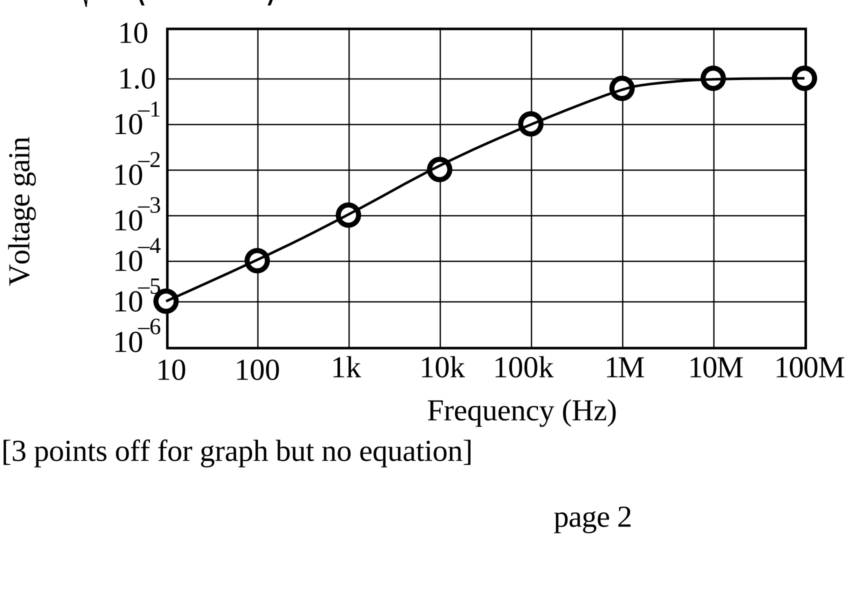
$$G_+ = G_{\pm} + G_c/2$$

If $V_- = 0$, then $V_0 = (G_{\pm} + G_c/2)V_+$

An alternative method would be to send V_{in} through inverting and a noninverting op-amp circuits to generate the pure differential inputs V_{in} and $-V_{in}$.

3b

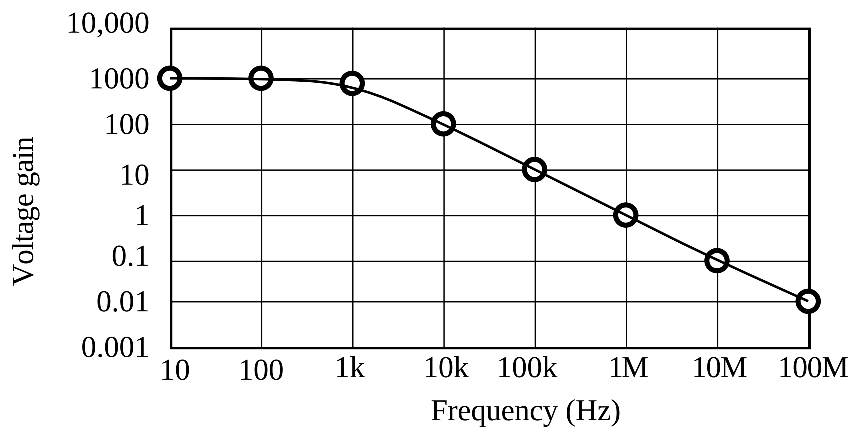
$$G_c = \frac{f/10^6\text{Hz}}{\sqrt{1+\left(f/10^6\text{Hz}\right)^2}}$$



[3 points off for graph but no equation]

3c

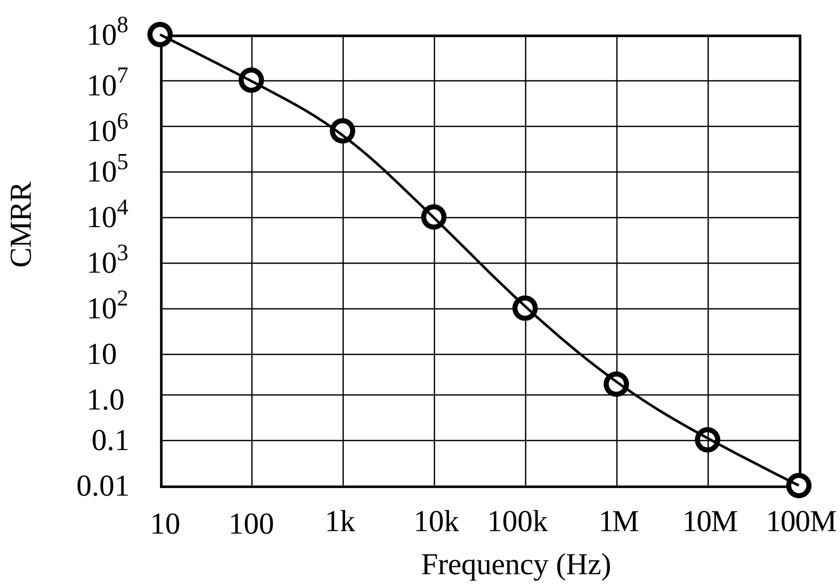
$$G_{\pm} = \frac{1000}{\sqrt{1 + \left(f / 10^3\,\text{Hz}\right)^2}}$$



[3 points off for graph but no equation]

3d

CMRR = $G_{\pm}/G_c$



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145L midterm #1 grade distribution:

Problem	
1	25.3 (30 max)
2	31.1 (35 max)
3	25.5 (35 max)

maximum score =	100	
average score =	81.9	
31-40	0	
41-50	2	D
51-60	1	C
61-70	0	
71-80	2	B–
81-90	5	B
91-95	0	
96-100	5	A