UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Evans Spring 1999

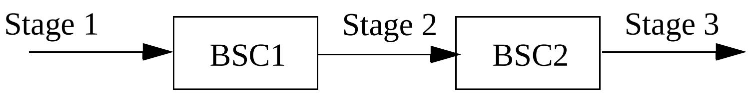
EECS 126 — Midterm #1

25 February 1999, 11:20 - 12:20 p.m.

Please explain clearly the steps in your answers.

Problem 1 (50 points)

Consider the cascade of two independent binary symmetric channels:



Let the error probabilities be ε_1 for BSC1 and ε_2 for BSC2. Suppose we generate a 0 or 1 randomly and input it to BSC1, observing the bit at all 3 stages.

Let B_i^x be the event that at stage i, an x is observed where $x \in \{0, 1\}$.

Assume $P(B_1^0) = P_0$ and that $P(B_1^x \cap B_3^y | B_2^z) = P(B_1^x | B_2^z) \cdot P(B_3^y | B_2^z)$ for $(x, y, z) \in \{0, 1\}^3$.

This says that B_1^x and B_3^y are conditionally independent given B_2^z.

Note that $P(B_2^0 | B_1^1) = P(B_2^1 | B_1^0) = \varepsilon_1$ and $P(B_3^0 | B_2^1) = P(B_3^1 | B_2^0) = \varepsilon_2$.

- **a)** Write down the sample space for this random experiment.
- **b)** Express the probability that we observe 0 at Stage 1, 1 at Stage 2, and 1 at Stage 3 in terms of the events $B_i^x, i \in \{1, 2, 3\}, x \in \{0, 1\}$.
- **c)** Show that $P(B_3^x | B_1^y \cap B_2^z) = P(B_3^x | B_2^z)$ using our conditional independence assumption.
- **d)** Calculate the probability in (b), above. (If you use the multiplication rule, (c) may be useful.)
- **e)** Calculate the probability that we observe a 0 at Stage 1 and a 1 at Stage 3.
- **f)** Calculate $P(B_3^1 | B_1^0)$.

Problem 2 (50 points)

Consider the following scheme for transmitting a data bit over a BSC.

1. Send the bit 3 times.
2. If 000 received, guess that 0 was transmitted.

   If 111 received, guess that a 1 was transmitted.

   Otherwise, request that the 3 bits be sent again.

Assume that errors occur independently every time the BSC is used, and that the error probability of the BSC is ε.

a) What is the probability that the group of 3 bits must be sent k times before a decision is made?

b) What is the probability that a decision is made after sending the group of 3 bits k times and it is the correct decision?

c) What is the probability that the correct decision is made?

d) Does this seem like an efficient scheme when $\varepsilon = 0.1$ say?

(Hint: You might like to consider the expected number of bits that needs to be sent to send each bit of actual data.)

Can you think of a simple scheme using the expected number of bits sent per data bit that has a lower error probability?